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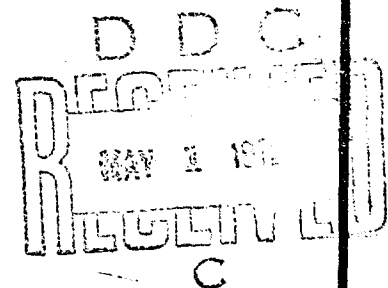
MEASURED WEIGHT, BALANCE, AND MOMENTS OF INERTIA OF THE X-24A LIFTING BODY

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Aerospace Research Engineer

TECHNOLOGY DOCUMENT No. 71-6

NOVEMBER 1971

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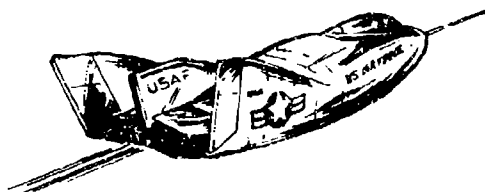
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FOREWORD

This technology document describes the measurement of the mass properties of the X-24A lifting body and presents a detailed summary of mass changes between and during test flights. The measurements were started on 28 November 1968, and analyses continued through the flight program which ended on 4 June 1971. Measurements were obtained at Edwards AFB at the AFFTC Weight and Balance Facility and at the NASA-FRC Heat Facility. References 1 through 8 are related documents which will be published.

The author wishes to acknowledge the contributions of Captain Johnny M. Ramoy, who performed all the initial analysis and Sergeant John C. Burch, who prepared and analyzed flight data. Acknowledgement is also extended to Mr. Chester H. Wolowicz of NASA-FRC who pioneered the inertia measurement technique and to other NASA-FRC personnel who assisted in the effort.

The participation of AFFTC personnel in this program was authorized by AFFTC Project Directive 69-38, and was performed under program structure 680A.

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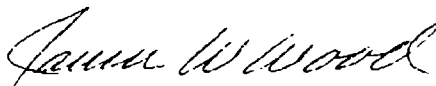
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ABSTRACT

Accurate values of weight, center of gravity, and moments of inertia were measured prior to the first flight of the X-24A lifting body. The weight, longitudinal, and lateral centers of gravity were measured at the AFFTC Weight and Balance Facility. The vertical center of gravity was measured by suspending the aircraft from a cable and determining the tilt angle as weights were added at the nose. Moments of inertia about each axis were measured by restraining the vehicle with springs and allowing it to vibrate about knife edges in the X- and Y-axes and a suspension cable in the Z-axis. These values were used as a baseline for mass data determination throughout the flight test program. A digital computer program was used to update the mass data for aircraft configuration changes and to produce time histories of mass data for powered flights, including the effects of rocket propellant flow and the changes in position of propellant in the tanks which result from accelerations on the aircraft.

table of contents

	Page
LIST OF ILLUSTRATIONS	vi
LIST OF TABLES	vii
LIST OF ABBREVIATIONS AND SYMBOLS	viii
INTRODUCTION	1
General	1
Approach	1
MEASUREMENT OF THE VERTICAL CENTER OF GRAVITY	2
Test Procedure	2
Computation Procedure	4
Vertical cg for the Inertia Measurement	10
MEASUREMENT OF I_x and I_{xz}	11
Test Procedure	11
Computation Procedure	15
Possible error in I_{xz} and	17
MEASUREMENTS OF I_x and I_y	18
Test Procedure	18
Computation Procedure	21
I_{table}	21
$I_{y_{t+c}}$ (pitch)	21
$I_{y_{t-c}}$ (roll)	22
Pitch	23
Roll	24

	Page
MOMENT OF INERTIA SUMMARY _____	25
DETERMINATION OF INFLIGHT WEIGHT AND BALANCE _____	26
Test Procedure _____	26
Computer Program _____	26
Addition and Subtraction Subroutines _____	26
Prime Subroutines _____	27
Propellant Angle Subroutines _____	27
Jettison Subroutine _____	28
Hydrogen Peroxide Subroutine _____	28
Landing Gear Effects _____	28
APPENDIX I - WEIGHT AND cg MEASUREMENTS _____	31
APPENDIX II - SPRING CALIBRATIONS _____	34
APPENDIX III - ITEMIZED LISTING OF COMPONENTS	
ADDED AND SUBTRACTED _____	42
APPENDIX IV - DERIVATION OF EQUATION 2 _____	46
APPENDIX V - X-24A MASS PROPERTIES COMPUTER	
PROGRAM LISTOUT _____	48
APPENDIX VI - TIME HISTORIES OF FLIGHT MASS PROPERTIES _____	61
APPENDIX VII - COMPARISON OF PREDICTED AND	
ACTUAL X-24A WEIGHTS _____	98
APPENDIX VIII - TRAPEZOIDAL APPROXIMATION OF PROPELLANT cg AS A	
FUNCTION OF PROPELLANT ANGLE _____	99
REFERENCES _____	100

list of illustrations

<u>Figure No.</u>	<u>Title</u>	<u>Page No.</u>
1	Vertical cg Measurement Apparatus _____	3
2	Definition of Terms in Equation 1 _____	5
3	Vertical Center of Gravity versus Aircraft Attitude _____	8
4	Schematic of Apparatus for Measurement of I_z _____	13
5	Apparatus for Measurement of I_z _____	14
6	Detail of Spring Attachments for I_z Measurement _____	14
7	Error in Measurement of I_{xz} _____	16
8	Platform Setup for Measurement of I_x and I_y _____	18
9	Setup for Measurement of I_y _____	19
10	Detail of Knife Edge _____	20
11	Springs Used for I_y Measurement _____	20
12	X-24A Variation in cg Position with Weight and and Propellant Angle _____	29
Appendix I		
1-3	X-24A Weight and Balance Records _____	31-33
Appendix II		
1-8	Spring Calibrations _____	34-41
Appendix IV		
1	Schematic of Aircraft Moments About the Knife Edge _____	46
Appendix VI		
1-18	Gross Weight, Center of Gravity, and Inertias _____	62-96

list of tables

<u>Table No.</u>	<u>Title</u>	<u>Page No.</u>
I	Weight Suspension at Nose, Gear Up _____	6
II	Weight Suspension at Nose, Gear Down _____	6
III	Vertical cg Computation, Gear Down _____	7
IV	Vertical cg Computation, Gear Up _____	7
V	Vertical cg - Gear Up (Flight 2 Configuration) _____	9
VI	Vertical cg - Gear Down (Flight 2 Configuration) _____	9
VII	Vertical cg - Gear Up (Inertia Test Configuration) _____	10
VIII	Vertical cg - Gear Down (Inertia Test Configuration) _____	11
IX	Vertical Center of Gravity Summary (With Respect to Water-Line Zero) _____	11
X	Computation Constants _____	15
XI	X-24A Moment of Inertia Summary _____	23
XII	X-24A Inertia Comparison with Predictions _____	25
XIII	X-24A Mass Properties Data (Flights X-1-2 through X-9-14 and Flight X-22-27) _____	30

Appendix III

I	Moment of Inertia Calculations _____	42
II	Inertial Subtractions from X-24A Prior to First Flight _____	43
III	Weight Removed Prior to First Flight _____	44
IV	Weight Added Prior to First Flight _____	44
V	Changes Between Flight 1 and Second Vertical cg Test (Flight X-2) _____	45
VI	Expendables Added Prior to Flight _____	45

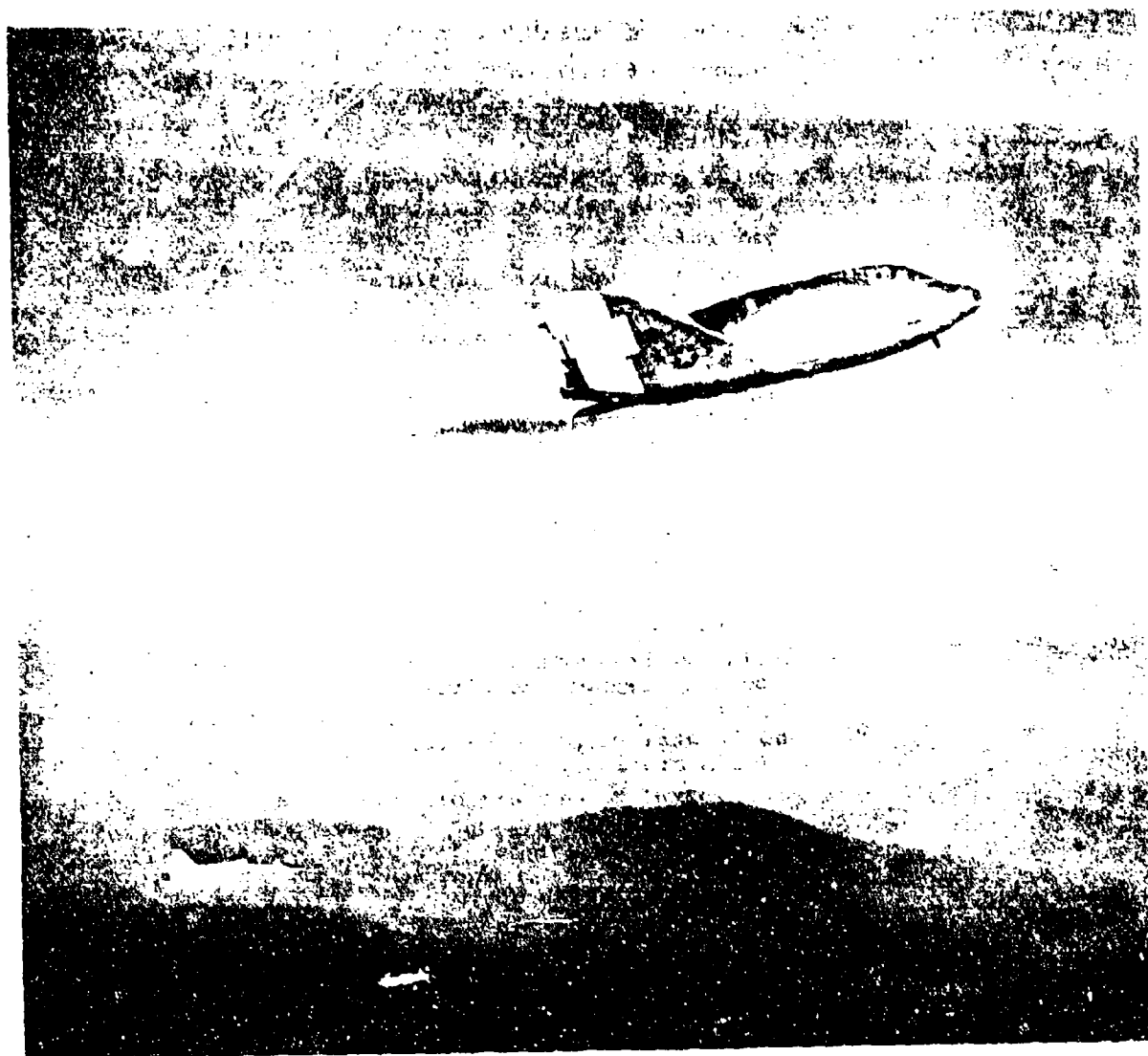
Appendix VIII

I	cg Location of Propellants _____	99
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list of abbreviations and symbols

<u>Item</u>	<u>Definition</u>	<u>Units</u>
a	moment arm of springs	ft
cg	center of gravity	in. or pct MAC
cg/($\theta_p = 0$)	cg along X-axis when propellant angle is zero	in.
Δcg	average distance between cg/($\theta_p = 0$) and cg with non-zero propellant angles	in.
g	acceleration due to gravity	32.2 ft/sec ²
h_c	height of X-24A cradle cg above knife edge	ft
h_{cg}	height of X-24A cg above knife edge	ft
h_t	height of inertia table cg above knife edge	ft
I_{table}	moment of inertia of inertia table about knife edge	slug-ft ²
I_{t+c}	moment of inertia for inertia table and X-24A cradle about knife edge	slug-ft ²
I_x	moment of inertia about X-axis	slug-ft ²
$I_{x,y} (KE)$	moment of inertia of X-24A about knife edge	slug-ft ²
$I_{x,y} body$	moment of inertia about X-24A body axes	slug-ft ²
$I_{x,y} combination (KE)$	moment of inertia of X-24A, table and X-24A cradle about knife edge	slug-ft ²
$I_{x,yt}$	moment of inertia of inertia table and cradle	slug-ft ²
I_{xz}	cross product of inertia	slug-ft ²
I_y	moment of inertia about Y-axis	slug-ft ²
I_z	moment of inertia about Z-axis	slug-ft ²
KE	knife edge	- - -
K_T	total spring constant	lb/ft
$K_{5,6,7,8} (large)$	spring constant of the four large springs	lb/ft
$K_{1,2} (small)$	spring constant of the two smaller springs	lb/ft
$K_{3,4} (medium)$	spring constant of the two medium springs	lb/ft
LOX	liquid oxygen	- - -

<u>Item</u>	<u>Definition</u>	<u>Units</u>
M	moments of inertia	slug-ft. ²
MAC	mean aerodynamic chord	in.
P	roll rate	deg/sec
PCM	pulse code modulation (used in the X-24A telemetry system)	- - -
R	yaw rate	deg/sec
RF	resultant body axis acceleration	g
W	weight of X-24A and experimental equipment	lb
w	incremental weight added to X-24A	lb
W _C	weight of X-24A table	lb
WL	waterline	- - -
W _T	weight of inertia table	lb
W _{X-24A}	weight of X-24A	lb
X _B	acceleration component along X-body axis	g
X _{cg}	relative horizontal motion of cg with respect to pivot point	in.
\bar{X}	horizontal cg measured from X-24A station 0.0	in.
X _w	horizontal moment arm of weight box	in.
\bar{Y}	lateral cg of X-24A	in.
\bar{Z}	vertical cg measured from WL = zero	in.
Z _B	acceleration component along Z-body axis	g
Z _{cg}	vertical cg measured from the pivot point	in.
Z _w	vertical moment arm of weight box	in.
δ	inclination of spring plane of action	deg
δ_0	inclination of spring plane of action at zero roll rate	deg
ϵ	inclination of principal aircraft axis	deg
θ	aircraft pitch attitude angle	deg
θ_p	propellant angle	deg
ω	frequency of oscillation	rad/sec



INTRODUCTION

GENERAL

An accurate knowledge of the center of gravity (cg) and moments of inertia is necessary for all dynamic analyses of aircraft, determination of stability derivatives from flight test data, and mechanization of accurate flight simulators. Contractor computations have been the prime source of such data for most test programs due to the lack of time, manpower, and equipment for making measurements, and the large size of modern aircraft. The validity of these computations depends on the accuracy of the individual component estimates and on the amount of time and effort spent in keeping track of changes made during the design and fabrication of the vehicle.

The requirements of the X-24A flight test program dictated that the cg and moments of inertia be determined experimentally. The first portion of this report describes the inertia measurement performed prior to the first flight of the aircraft. This measurement first used a vertical cg obtained at NASA-FRC using the suspension method. When discrepancies were discovered between the vertical cg from the suspension method and the vertical cg determined at the weight and balance facility, the suspension test was repeated using improved procedures. This report includes corrections resulting from the later vertical cg measurement performed prior to the second flight. Test theory, procedures, equipment, data reduction techniques, and results are discussed in this report.

These measured mass data (weight, cg, and moments of inertia) were used as a baseline for all later weight and balance determinations during the flight test program. The weight, cg, and moments of inertia were recomputed for each weight change of the aircraft. Weight and horizontal cg locations were correlated each time the aircraft was weighed. The second portion of this report presents the mass data for each flight, including time histories for powered flights which incorporated the effects of propellant utilization and propellant angle on the aircraft cg.

APPROACH

The ultimate objective of this effort was an accurate determination of the body axis cg, moments of inertia, and weight. The body axis system used was a standard right-hand orthogonal system with its origin at the cg. Since the moments of inertia in roll and pitch were measured about an axis of rotation that was parallel, but displaced from the body axis, the longitudinal, vertical, and lateral cg's were required in order to transfer the measured inertias from the axis of rotation to the body axes. The inclination of the principal axis was also required in order to compute the cross product of inertia, I_{xz} . Two techniques were used to determine the vertical cg. The first involved tilting the vehicle on a weighing platform at the AFFTC Weight and Balance Facility and recording variations in nose gear and main gear reactions with tilt angle. The second suspended the vehicle at the NASA-FRC Heat Facility from a single cable and recorded variations in tilt angle as known weights were applied at the nose and tail. The latter technique was felt to be the more accurate and is described in detail in this report. Measurements of the moment of inertia about the Z-axis (I_z) and the inclination of the principal aircraft axis (ϵ) were accomplished by suspending the vehicle

from a single cable, restraining vehicle yaw with calibrated springs and recording the oscillatory characteristics. The measurement of the moments of inertia about the X and Y axes (I_x and I_y) was accomplished by balancing the vehicle on two knife edges, restraining it in either the pitch or roll axis with calibrated springs, and recording the oscillatory characteristics. Each test and the subsequent calculations of moments of inertia are discussed separately.

The ground test values of mass data were used as a baseline for the subsequent flight test program. A digital computer program was written to account for vehicle weight changes which occurred between flights as well as changes in expendable quantities, such as rocket engine propellants, which varied during flight. Wherever possible, ground test results or inflight measurements were used to calculate the rate of use of the expendables.

MEASUREMENT OF THE VERTICAL CENTER OF GRAVITY

TEST PROCEDURE

Prior to the measurement of the X-24A moments of inertia in November 1968, the weight and longitudinal, lateral, and vertical cg's were measured at the AFFTC Weight and Balance Facility.

In order to get mass data which would be representative of the first glide flight, ballast was added to the rear of the aircraft to obtain the desired cg (58.5 percent MAC). The ballast locations for these tests was not yet permanent. This necessitated removing the weights mathematically from the experimentally determined measurements and then adding them back after the final ballast installation was defined. A dummy pilot and parachute was also on board during the measurement. The vehicle was weighed at attitudes ranging from approximately -16 degrees (nosedown) to +12 degrees (noseup). Knowing the reactions at the nose gear and main gear along with the geometry of the weighing scale, the gear-down vertical cg was determined. The results obtained are shown in appendix I.

The vehicle weight and horizontal cg obtained from this weighing were used for subsequent calculations, however, the measurement of the vertical cg was not considered reliable because of the small range of vehicle attitude angles obtained and potential errors in measured dimensions due to landing gear bending. A separate vertical cg measurement was made at NASA-FRC by suspending the vehicle to obtain a more reliable value of both the gear-up and gear-down vertical cg's. As a comparison, the gear-down vertical cg obtained at the Weight and Balance Facility was 23.86 inches above waterline (WL) zero, and the value from the vertical suspension measurement was 24.36 inches. More confidence was placed in the latter value, and it was used as a baseline for all calculations in this report. The more accurate vertical suspension measure was performed several months after the inertia swing, just prior to the second flight, when errors in the first vertical suspension were discovered. At that time, the aircraft had different mass properties.

The second measurement is described, and the result is corrected mathematically to obtain a vertical cg for the airplane at the time of the inertia swing. This corrected vertical cg was used for all subsequent moment of inertia calculations.

The X-24A was suspended with a single cable from the overhead crane in the NASA-FRC Heat Facility. The cable, proof-tested to 20,000 pounds, was attached to the X-24A hoisting bar, and the bar was attached to the vehicle at the same points used for mating with the B-52 pylon adapter (figure 1).

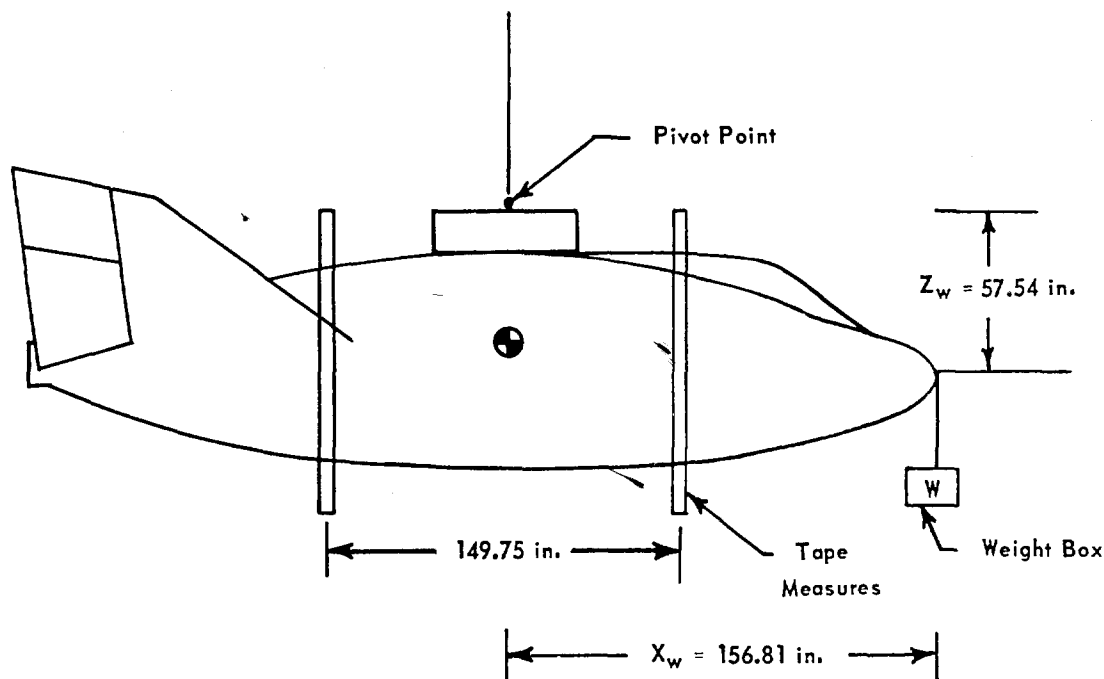


Figure 1 Vertical cg Measurement Apparatus

Special care was exercised to insure that the vehicle could only rotate in pitch about a single pivot point (the center of the bolt of the hoist eye) by wrapping the hoist eye with wire. The location of the pivot point was accurately measured. Bags of lead shot were added to the nose to level the vehicle. A transit was used to sight the vehicle attitude by reading two metal tape measures attached at reference points on the fuselage. Known weights were added to a weight box mounted on the aircraft noseboom, and vertical center of gravity was calculated from the displacement of the vehicle's position. The measurement was made both gear-up and gear-down. After adding each weight, the vehicle was allowed to stabilize and the tape measures at each level point were read using the transit. The vehicle attitude for each weight was determined from the readings of the front and rear tapes and knowledge of the distance between the tapes. Using Equation 1, the vertical cg relative to the suspension point was determined using the vehicle attitude and the vertical and horizontal arms of each known weight.

Theoretically, only one attitude was required to determine the vertical cg. For accuracy, however, several weights were added to the box mounted on the noseboom, and tape measure sightings were recorded. The weights were removed in the reverse order of that in which they were put on to provide a hysteresis check. The same procedure was repeated with the weight box suspended from the engine mount at the rear of the vehicle.

COMPUTATION PROCEDURE

Computation of the vertical cg was accomplished using Equation 1, the terms of which are defined in figure 2. Equation 1 is derived as follows (reference 9):

$$X_w'' = X_w \cos \theta - Z_w \sin \theta$$

$$\Delta X_{cg} = Z_{cg} \sin \theta$$

Summing moments, $\Sigma M_{pivot} = 0$

$$wX_w'' = W(\Delta X_{cg}) = WZ_{cg} \sin \theta$$

$$Z_{cg} = \frac{w}{W} \left(\frac{X_w \cos \theta - Z_w \sin \theta}{\sin \theta} \right)$$

$$Z_{cg} = \frac{w}{W} \left(\frac{x_w}{\tan \theta} - Z_w \right) \quad (1)$$

where

w = incremental weight added

W = weight of X-24A and experimental equipment

X_w = horizontal lever arm of the weight box

Z_w = vertical level arm of the weight box

$$\theta = \tan^{-1} \left(\frac{\text{net change in tape readings}}{\text{distance between tapes}} \right)$$

A tangent function resulted since the tape measures were attached to, and therefore rotated with, the vehicle. The total weight was determined as follows:

Total weight (gear down)

X-24A (less pilot)	5,927.0
Hoist beam	453.0
Shot bags to level	58.4
	<u>6,438.4</u> pounds

Total weight (gear up)

X-24A (less pilot)	5,927.0
Hoist beam	453.0
Shot bags to level	37.2
	<u>6,417.2</u> pounds

The distance between the tapes was

149.75 inches

Distance from pivot point to weights suspended at nose:

Horizontal distance: 156.81 inches

Vertical distance: 57.54 inches

Distance from pivot point to water-line zero: 90.337 inches

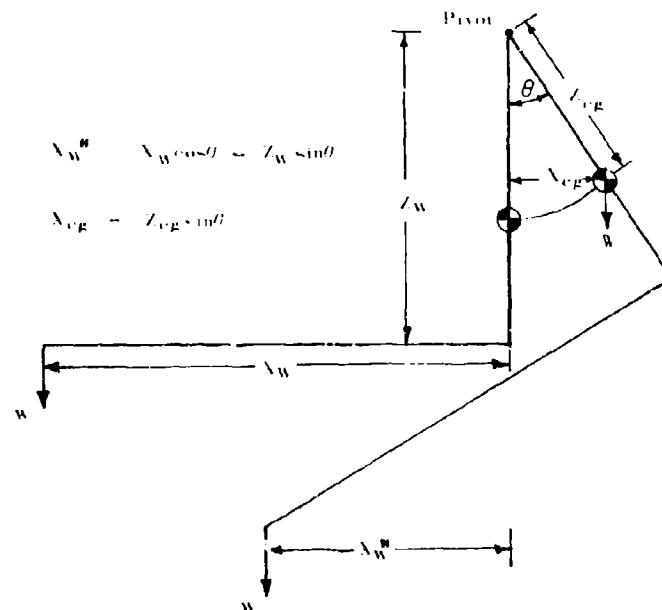


Figure 2 Definition of Terms in Equation 1

The tape measure readings for the gear-up and gear-down configurations are summarized in tables I and II. The net relative change in tape reading is used in tables III and IV to determine a vertical cg for each incremental weight.

The computed vertical cg's for each aircraft attitude from table IV are plotted in figure 3. The Zcg values obtained at larger angles were weighted more heavily when a line was faired through the data, since less error is introduced in computing $\tan \theta$ at these angles. The values used for the vertical distance from the suspension point to the cg were 62.75 inches for gear down and 60.00 inches for gear up.

TABLE I
Weight Suspension at Nose - Gear Up

Weight	Front Tape			Rear Tape			Relative Change in Tape Reading			
	Incr	Decr	Avg	Incr	Decr	Avg	Front	Rear	Net	Net
0	$51\frac{8}{16}$	$51\frac{8}{16}$	$51\frac{16}{32}$	$51\frac{10}{16}$	$51\frac{10}{16}$	$51\frac{20}{32}$	0	0	0	0
58.0	$48\frac{5}{16}$	$48\frac{5}{16}$	$48\frac{10}{32}$	$51\frac{15}{16}$	$51\frac{15}{16}$	$51\frac{30}{32}$	$3\frac{3}{32}$	$-\frac{10}{32}$	$3\frac{16}{32}$	3.500
108.1	$45\frac{10}{16}$	$45\frac{9}{16}$	$45\frac{19}{32}$	$52\frac{7}{32}$	$52\frac{8}{32}$	$52\frac{7}{32}$	$5\frac{29}{32}$	$-\frac{19}{32}$	$6\frac{16}{32}$	6.500
158.3	$43\frac{1}{16}$	$43\frac{1}{16}$	$43\frac{2}{32}$	$52\frac{17}{32}$	$52\frac{17}{32}$	$52\frac{17}{32}$	$8\frac{14}{32}$	$-\frac{30}{32}$	$9\frac{12}{32}$	9.375
208.0	$40\frac{9}{16}$	$40\frac{14}{32}$	$40\frac{16}{32}$	$52\frac{29}{32}$	$52\frac{29}{32}$	$52\frac{29}{32}$	11	$-\frac{9}{32}$	$12\frac{9}{32}$	12.202
258.0	$38\frac{1}{16}$	$38\frac{1}{32}$	$38\frac{2}{32}$	$53\frac{4}{16}$	$53\frac{9}{32}$	$53\frac{8}{32}$	$13\frac{14}{32}$	$-\frac{20}{32}$	$15\frac{2}{32}$	15.063
306.3	$35\frac{23}{32}$	$35\frac{11}{16}$	$35\frac{22}{32}$	$53\frac{11}{16}$	$53\frac{23}{32}$	$53\frac{23}{32}$	$15\frac{26}{32}$	$-\frac{3}{32}$	$17\frac{29}{32}$	17.906
356.4	$33\frac{12}{32}$	$33\frac{13}{32}$	$33\frac{13}{32}$	$54\frac{1}{16}$	$54\frac{1}{16}$	$54\frac{2}{32}$	$18\frac{3}{32}$	$-\frac{14}{32}$	$20\frac{17}{32}$	20.532

TABLE II
Weight Suspension at Nose - Gear Down

Weight	Front Tape			Rear Tape			Relative Change in Tape Reading			
	Incr	Decr	Avg	Incr	Decr	Avg	Front	Rear	Net	Net
0	$51\frac{11}{16}$	$51\frac{10}{16}$	$51\frac{21}{32}$	$51\frac{11}{16}$	$51\frac{9}{16}$	$51\frac{20}{32}$	0	0	0	0
58.0	$49\frac{2}{16}$	$48\frac{15}{16}$	$49\frac{3}{32}$	$51\frac{14}{16}$	$51\frac{29}{32}$	$51\frac{28}{32}$	$2\frac{20}{32}$	$-\frac{8}{32}$	$2\frac{28}{32}$	2.875
108.1	$46\frac{2}{16}$	46	$46\frac{1}{32}$	$52\frac{7}{32}$	$52\frac{3}{16}$	$52\frac{7}{32}$	$5\frac{20}{32}$	$-\frac{19}{32}$	$6\frac{7}{32}$	6.217
158.3	$43\frac{10}{16}$	$43\frac{6}{16}$	$43\frac{16}{32}$	$52\frac{17}{32}$	$52\frac{16}{32}$	$52\frac{16}{32}$	$8\frac{5}{32}$	$-\frac{28}{32}$	$9\frac{1}{32}$	9.031
208.0	$41\frac{7}{32}$	$41\frac{1}{16}$	$41\frac{5}{32}$	$52\frac{29}{32}$	$52\frac{14}{16}$	$52\frac{28}{32}$	$10\frac{16}{32}$	$-\frac{8}{32}$	$11\frac{24}{32}$	11.750
258.0	$38\frac{13}{16}$	$38\frac{10}{16}$	$38\frac{23}{32}$	$53\frac{4}{16}$	$53\frac{3}{16}$	$53\frac{7}{32}$	$12\frac{30}{32}$	$-\frac{19}{32}$	$14\frac{17}{32}$	14.531
306.3	$36\frac{15}{32}$	$36\frac{12}{32}$	$36\frac{13}{32}$	$53\frac{17}{32}$	$53\frac{7}{16}$	$53\frac{17}{32}$	$15\frac{8}{32}$	$-\frac{29}{32}$	$17\frac{7}{32}$	17.156
356.4	$34\frac{3}{16}$	$34\frac{3}{16}$	$34\frac{6}{32}$	$53\frac{15}{16}$	$53\frac{15}{16}$	$53\frac{30}{32}$	$17\frac{15}{32}$	$-\frac{10}{32}$	$19\frac{25}{32}$	19.781

The vertical cg with respect to water-line zero was found by subtracting the values of z_{cg} from the total distance of the pivot point to the water-line zero. The gear up and gear down measured values are shown below.

Gear up: $90.337 - 60.00 = 30.337$ inches

Gear down: $90.337 - 62.75 = 27.787$ inches

TABLE III
Vertical cg Computation, Gear Down

$$z_{cg} = \frac{w}{W} \left(\frac{x_w}{\tan \theta} - z_w \right)$$

W 6417.2

$x_w = 156.81$

$z_w = 57.54$

w	Net Hel Tape Chan.	Tan $\theta =$ Net 149.75	θ	$\frac{156.81}{\tan \theta}$	(A) - 57.54	$z_{cg} = \frac{w}{W}$ (B)
58.0	2.875	.0192	1.0999	8167.7557	8110.2157	73.3018
108.1	6.217	.0415	2.3781	3775.8960	3718.3560	62.6370
158.3	9.051	.0603	3.4519	2599.6125	2542.0725	62.7980
208.0	11.750	.0785	4.4865	1998.4934	1940.9534	62.9099
258.0	14.531	.0970	5.5432	1616.0139	1558.4739	62.6576
306.3	17.156	.1146	6.5356	1368.7513	1311.2113	62.5856
356.4	19.781	.1321	7.5252	1187.0538	1129.5138	62.7312

TABLE IV
Vertical cg Computation, Gear Up

W 6438.1

58.0	3.500	.0234	1.3389	6709.2279	6651.6879	59.9214
108.1	6.500	.0424	2.4854	3612.6612	3555.1212	59.6901
158.3	9.375	.0626	3.5823	2504.7784	2447.2384	60.1699
208.0	12.282	.0820	4.6887	1911.9278	1854.3878	59.9082
258.0	15.063	.1006	5.7439	1558.9390	1501.3990	60.1642
306.3	17.906	.1196	6.8186	1311.4206	1253.8906	59.6520
356.4	20.532	.1371	7.8071	1143.6927	1086.1527	60.1244

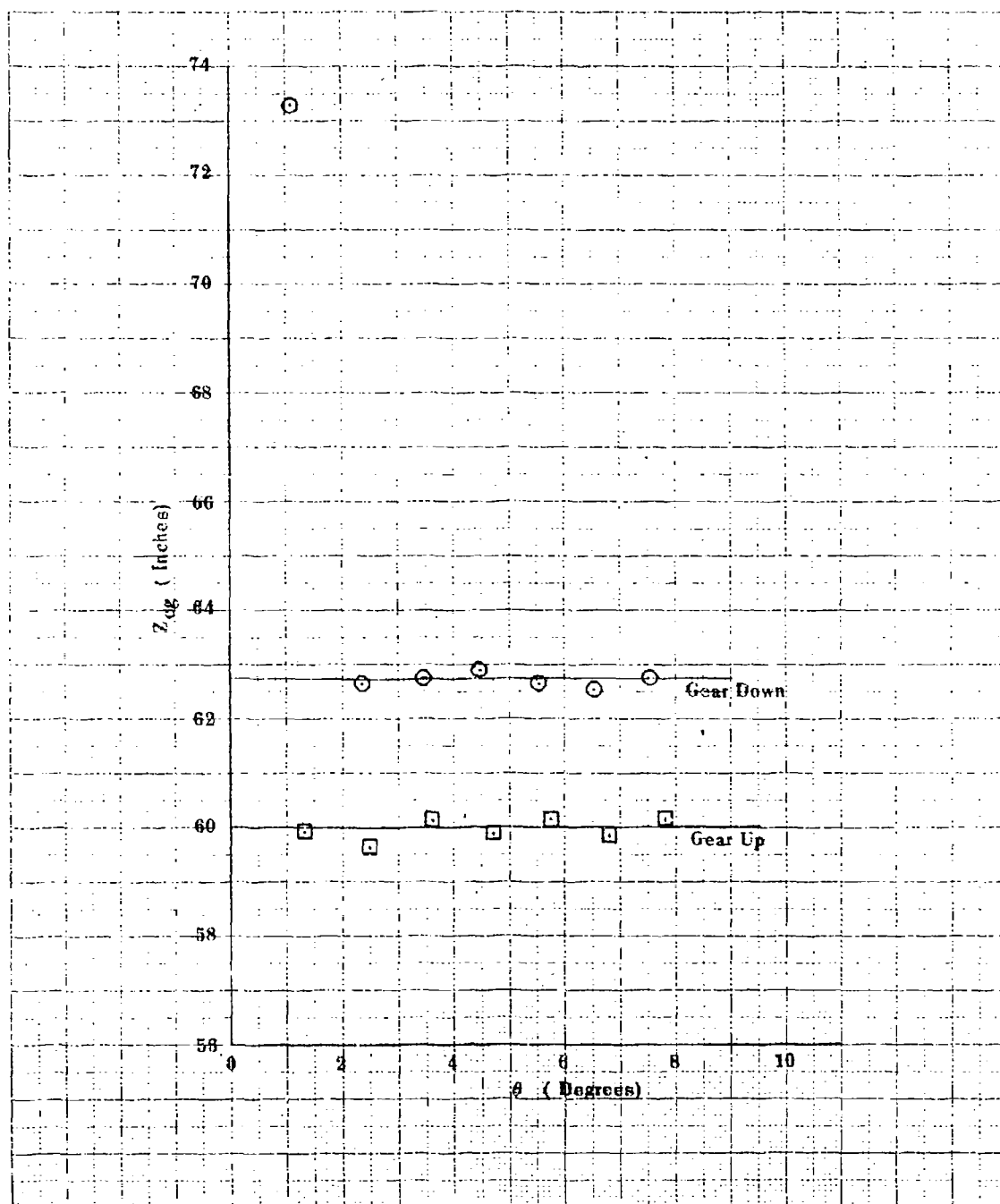


Figure 3 Vertical Center of Gravity versus Aircraft Attitude

This measured vertical cg includes the mass effects of the hoist bar and balance weights. The moments that these items created about the suspension pivot point are removed in tables V and VI to obtain the gear up and gear down conditions for the empty X-24A (flight 2 configuration).

Table V

VERTICAL cg - GEAR UP (FLIGHT 2 CONFIGURATION)

Item	Weight (lb)	Vertical Displacement (in.)	Moment (in.-lb)
X-24A total	6,438.4	60.00	386,304.000
Hoist bar	-453.0	17.187	-7,785.711
balance weights	-58.4	49.35	-2,892.040
X-24A	5,927.0	63.377	375,636.249
With respect to water-line zero: $\bar{Z} = 90.337 - 63.377 = 26.96$ inches			

Table VI

VERTICAL cg - GEAR DOWN (FLIGHT 2 CONFIGURATION)

Item	Weight (lb)	Vertical Displacement (in.)	Moment (in.-lb)
X-24A total	6,417.2	62.75	402,679.300
Hoist bar	-453.0	17.187	-7,735.711
Balance weights	-37.2	49.35	-1,835.820
X-24A	5,927.0	66.316	393,057.769
With respect to water-line zero: $\bar{Z} = 90.337 - 66.316 = 24.02$ inches			

VERTICAL cg FOR THE INERTIA MEASUREMENT

In order to determine the X-24A moments of inertia, a new test vertical cg had to be determined for the test measurement aircraft configuration (including ballast weights, dummy pilot, less flight weights (permanent ballast), and changes between flights). This was the vertical cg used to obtain the moment of inertia about the knife edge and to transfer the inertias to the body axes. The moments and weights of items added and subtracted from the aircraft are computed in appendix III for the suspension point used in this second measurement. The moments due to changes between the first and second flights are also accounted for.

This vertical cg was used for computation of all moment of inertia transfers from the inertia measurement. The gear down vertical cg for the test configuration was also obtained.

Table VII

VERTICAL cg - GEAR UP (INERTIA TEST CONFIGURATION)

Item	Weight (lb)	Moment (in.-lb)	Vertical Displacement (in.)
X-24A	5,927.0	375,636.249	63.377
Ballast weights, dummy pilot hoist eyes, ballast box	+616.416	+39,732.537	- - -
Flight weight	-154.0	-10,831.898	- - -
Changes from third flight	-9.54	-1,034.58	- - -
X-24A at inertia measurement	6,379.88	403,502.308	63.246
With respect to water-line zero:			
$\bar{z} = 90.337 - 63.246 = 27.09$ inches			

Table VIII

VERTICAL cg - GEAR DOWN (INERTIA TEST CONFIGURATION)

Item	Weight (lb)	Vertical Displacement (in.)	Moment (in.-lb)
X-24A	5,927.0	66.316	393,057.769
Ballast, etc.	+616.416	- - -	39,732.537
Flight weight	-154.0	- - -	-10,831.898
Changes	-9.54	- - -	-1,034.58
X-24A at test	6,379.88	65.977	420,923.828
With respect to water-line zero: $\bar{Z} = 90.337 - 65.977 = 24.36$ inches			

Table IX

VERTICAL CENTER OF GRAVITY SUMMARY (WITH RESPECT TO WATER-LINE ZERO)

Configuration	\bar{Z} Gear-Up	\bar{Z} Gear-Down
1. Measured value, suspended X-24A including hoist bar & balance weights	30.337	27.787
2. Corrected for removal of hoist bar & balance weight	26.95	24.02
3. Corrected to X-24A configuration at time of inertia swing	27.09	24.36

MEASUREMENT OF I_z and I_{xz}

TEST PROCEDURE

To obtain the moment of inertia about the Z-axis and the inclination of the principal axis, the X-24A was suspended from a single cable with the landing gear retracted. One end of the cable was attached to an overhead crane by a swivel to minimize torsional effects, and the other end was attached to the X-24A hoist bar approximately above the aircraft cg (figures 4 and 5). Sixty pounds of lead shot were added at the rear to level the vehicle.

A lightweight aluminum channel was bolted to the jackpads of the vehicle and was braced longitudinally with another aluminum channel. Four springs, two per side, were attached to the channel as close to the

aircraft longitudinal cg location as possible. Figures 4 through 6 show that this insures a 90-degree angle between the channel and the spring line of action. The channel was 12 feet long, giving a lever arm of 6 feet for each pair of springs. The springs were attached through an eye bolt at the end of channel (figures 4 and 5). Both the channel bracing and securing of the eye bolt were necessary to prevent secondary spring constants. The springs were then connected to vertical tiebacks through lightweight tubing and turnbuckles. The tiebacks each had 17 holes, the middle hole on each being used to insure a level plane of action for all 4 springs. The turnbuckles allowed all four springs to be preloaded to insure operation in their linear range. The spring calibrations are shown in appendix 11.

Three different sets of springs were used. The smaller springs were selected because of their light weight, which minimized the sag in the tie-back apparatus and gave a straight line of action for the springs. For the measurement of I_z and I_{xz} , one small and one medium spring were used on each side, which presented preloading problems. The springs had a linear range of eight inches, but had to be preloaded two inches to insure operation in this range.

After the vehicle was leveled and the springs preloaded, pressure was applied horizontally to the pitot boom until the X-24A began oscillating at the proper test amplitude. Care was taken in starting the motion to insure that yawing motion and not pitching motion was induced and that the springs were not stretched out of their linear range at the high or low end. Thus, sag was eliminated and oscillation amplitudes were kept as low as possible. Several trial runs were made and the setup was inspected to insure that all secondary spring constants had been eliminated since these can cause a change in frequency with time, causing a beat to occur between the yaw and roll motions.

The vehicle instrumentation was used to record yaw, roll, and pitch rates. The X-24A PCM system transmitted test data to NASA's mobile telemetry van parked near the aircraft for recording and display. The yawing forcing function was applied to the boom and the vehicle was allowed to oscillate freely for a few seconds before the data were recorded. Three test runs were made and the frequencies were averaged. A stopwatch was used as a backup to the instrumentation.

I_{xz} and ϕ were measured using the same setup as described for I_z . Since the amount of roll rate induced by a pure yawing moment is directly proportional to ϕ and I_{xz} , these two quantities can be determined by applying a pure yawing moment to the aircraft at different pitch attitudes and measuring the amplitude ratio of roll rate to yaw rate. The aircraft attitude where roll rate to yaw rate (P/R) is zero defines the inclination of the principal axis.

Instead of producing aircraft motion at different pitch attitudes, the inclination of the spring plane of action was varied by moving the tie-back points up and down using the holes shown in figures 4 through 6. The tie-back was moved up two holes for the two forward springs and down two holes for the rear springs. The aircraft was again disturbed and the roll rate and yaw rate recorded in the mobile van. This procedure was repeated at several different angles of spring action for pitch up and down to check for hysteresis and to insure that the inclina-

tion of the principal axis had been passed through (figure 4). The springs were pre-stressed at each point to insure linearity and to check that the line of action was straight. Knowing the inclination of the spring plane of action and the roll rate to yaw rate amplitude ratio, c was determined. This procedure for measuring I_{xz} and c is discussed at length in reference 9.

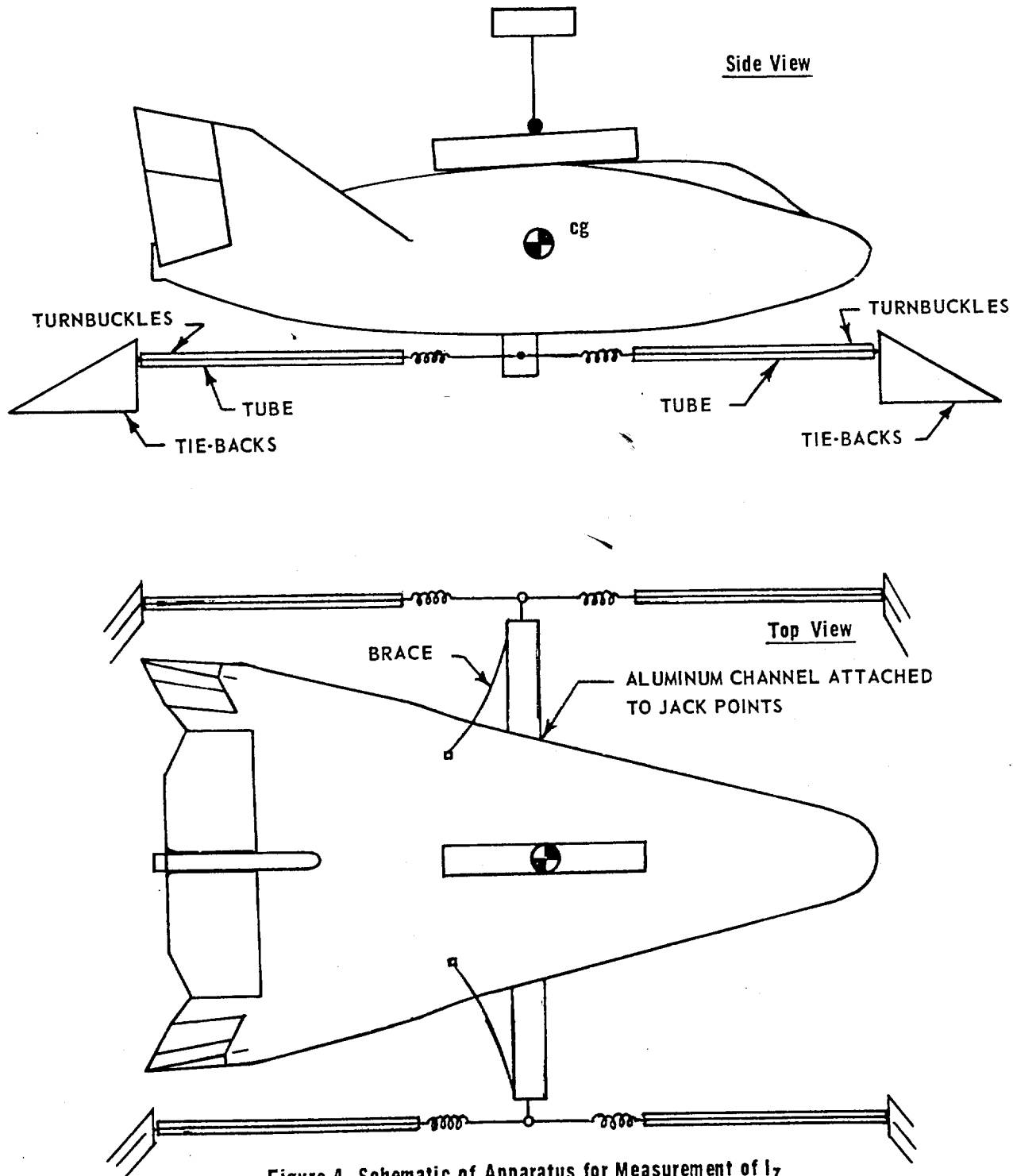


Figure 4 Schematic of Apparatus for Measurement of I_z

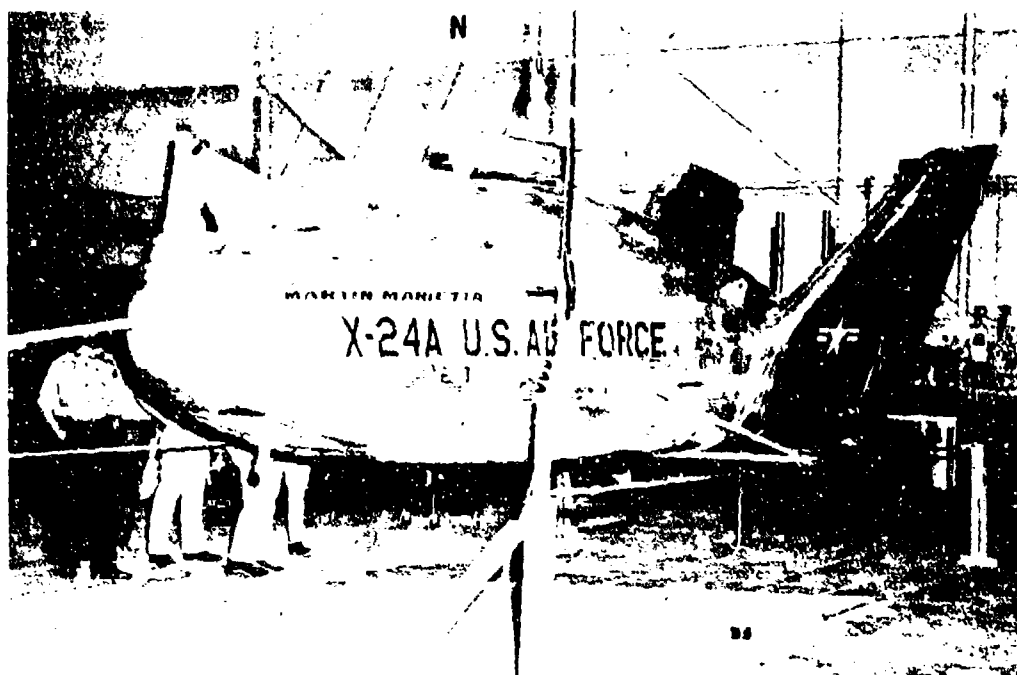


Figure 5 Apparatus for Measurement of I_z

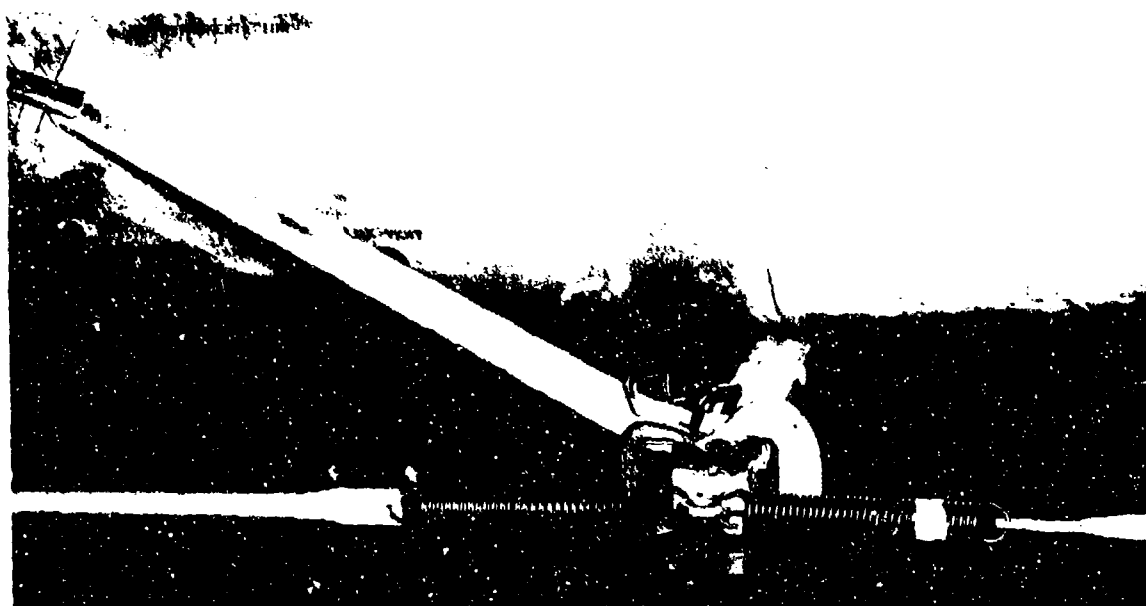


Figure 6 Detail of Spring Attachment for I_z Measurement

COMPUTATION PROCEDURE

For convenience, the constants used in computation of all moments of inertia are presented in table X.

Table X
COMPUTATION CONSTANTS

Weight and Balance Constants	Spring Constants
$l = 6 \text{ ft}$ $W_T = 965 \text{ lb}$ $W_C = 635 \text{ lb}$ $h_T = 3.5 \text{ in.} = 0.2915 \text{ ft}$ $h_C = 10.5 \text{ in.} = 0.875 \text{ ft}$ $W_{X-24A} = 6,380 \text{ lb}$ $h_{cg} = 44.606 \text{ in.} = 3.717 \text{ ft}$ $M = 198.14 \text{ slugs}$	<u>Small Springs</u> $K_1 = 120.8 \text{ lb/ft}$ $K_2 = 116.9 \text{ lb/ft}$ <u>Medium Springs</u> $K_3 = 255.70 \text{ lb/ft}$ $K_4 = 259.70 \text{ lb/ft}$ <u>Large Springs</u> $K_5 = 767.16 \text{ lb/ft}$ $K_6 = 762.34 \text{ lb/ft}$ $K_7 = 764.48 \text{ lb/ft}$ $K_8 = 766.32 \text{ lb/ft}$ <u>Spring Constant Totals</u> $K_T \text{ (for } I_z) = K_1 + K_2 + K_3 = 755.1 \text{ lb/ft}$ $K_T \text{ (for } I_{table}) = K_3 + K_4 = 515.4 \text{ lb/ft}$ $K_T \text{ (for } I_y \text{ and } I_x) = K_5 + K_6 + K_7 + K_8 = 3061.3 \text{ lb/ft}$

The equation for determining I_z is:

$$I_z = \frac{K_T a^2}{\omega^2} \quad (2)$$

The measured frequency was:

$$\omega = 1.697 \text{ rad/sec (average of 3 runs)}$$

using constants from table X:

$$I_z = 9,440 \text{ slug-ft}^2$$

This measured value of I_z contains the inertial contributions of the hoist bar, shot bags, and spring attachment apparatus. These must be subtracted to obtain the X-24A body axis moment of inertia. In addition, the flight weights were added to obtain the empty aircraft inertias. These changes are itemized in appendix III.

$$\begin{aligned}
 I_{z_{\text{first flight}}} &= I_{z_{\text{measured}}} - I_{z_{\text{subtracted}}} + I_{z_{\text{added}}} \\
 (\text{empty aircraft}) &= 9,440 - 946.27 + 418.7 \\
 &= 8,912 \text{ slug-ft}^2
 \end{aligned}$$

From reference 9, the equation for relating the inclination of the spring plane of action (ϕ) to I_{xz} is:

$$I_{xz} = I_z \tan \phi_0$$

where:

ϕ_0 = inclination of springs plane of action at zero roll rate to yaw rate ratio

Figure 7 shows a graph of the tangent of the inclination angle versus roll rate to yaw rate ratio. It shows ϕ to be equal to zero for a zero P/R ratio. This was verified visually by noting that for the level position no roll rate occurred as a result of yawing motion. Thus, the measured value for I_{xz} was zero. The quality of the telemetered roll and yaw rate data was poor for this test and an error analysis was made.

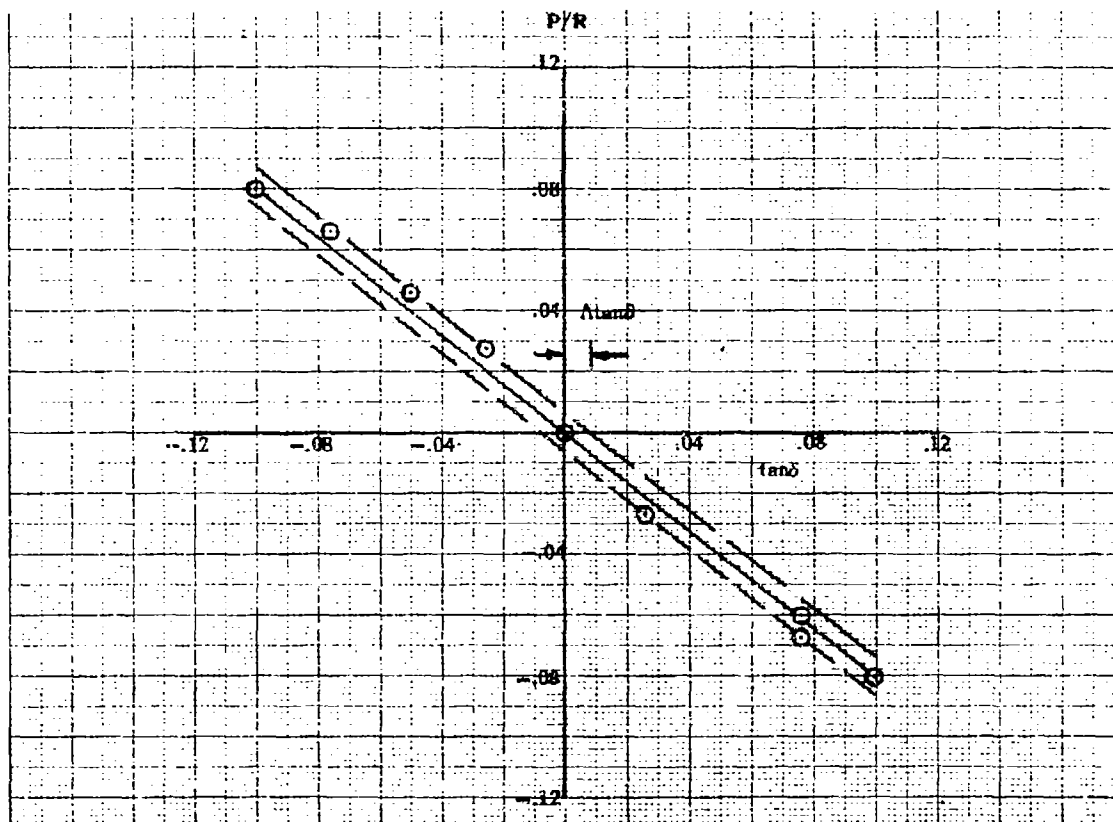


Figure 7 Error in Measurement of I_{xz}

Of the several possible sources of error, the error in ϵ was due more to noise in the measured values of P and R than errors in the measurement of spring tie-back points or the distance between tie-backs since these are direct measurements. A band is shown in figure 7 that encloses the total error observed from measuring P/R.

POSSIBLE ERROR IN I_{xz} AND :

$$\Delta I_{xz} = I_z \Delta \tan \epsilon$$

$$\Delta \tan \epsilon = \pm 0.008 \text{ (from figure 6)}$$

$$I_z = 8,912 \text{ slug-ft}^2$$

$$I_{xz} = 0 \text{ slug-ft}^2 \text{ (measured)}$$

$$\Delta I_{xz} = \pm (8,912)(0.008)$$

$$\Delta I_{xz} = \pm 71.29 \text{ slug-ft}^2$$

The measured value for I_{xz} is then

$$I_{xz} = 0 \pm 71.29 \text{ slug-ft}^2$$

Using the inertia increments from appendix III, the calculated first flight value for I_{xz} is:

$$\begin{aligned} I_{xz} &= I_{xz \text{ measured}} - I_{xz \text{ subtracted}} + I_{xz \text{ added}} \\ &= (0 \pm 71.29) - (94.35) + (-14.59) \\ &= 79.76 \pm 71.29 \text{ slug-ft}^2 \end{aligned}$$

The inclination of the principal axis to the body axis can be related to I_z , I_x , and I_{xz} . This expression (Equation 3) is given in reference 9.

$$\epsilon = \frac{1}{2} \tan^{-1} \left(\frac{2 I_{xz}}{I_z - I_x} \right) \quad (3)$$

The first flight value of ϵ was calculated for the bound of I_{xz} and measured values of I_z and I_x .

For

$$I_{xz} = 79.76 \pm 71.29 \text{ slug-ft}^2$$

$$\epsilon = 0.617 \pm 0.56 \text{ degrees}$$

MEASUREMENTS OF I_x and I_y

TEST PROCEDURE

The moments of inertia for roll and pitch were measured by mounting the X-24A on its shipping cradle and the cradle on a platform especially constructed for determining the moments of inertia. The platform was constructed of 6-inch steel I-beams and had a total weight of 965 pounds (figure 8). The shipping crate material was 4x4 inch hardwood. The hardwood shipping cradle was attached to the moment of inertia platform by an 8-inch steel channel on one end and a 6-inch I-beam on the other. The cradle and attachments had a total weight of 635 pounds. Figure 9 shows X-24A, cradle, steel channel, and I-beam attachments.

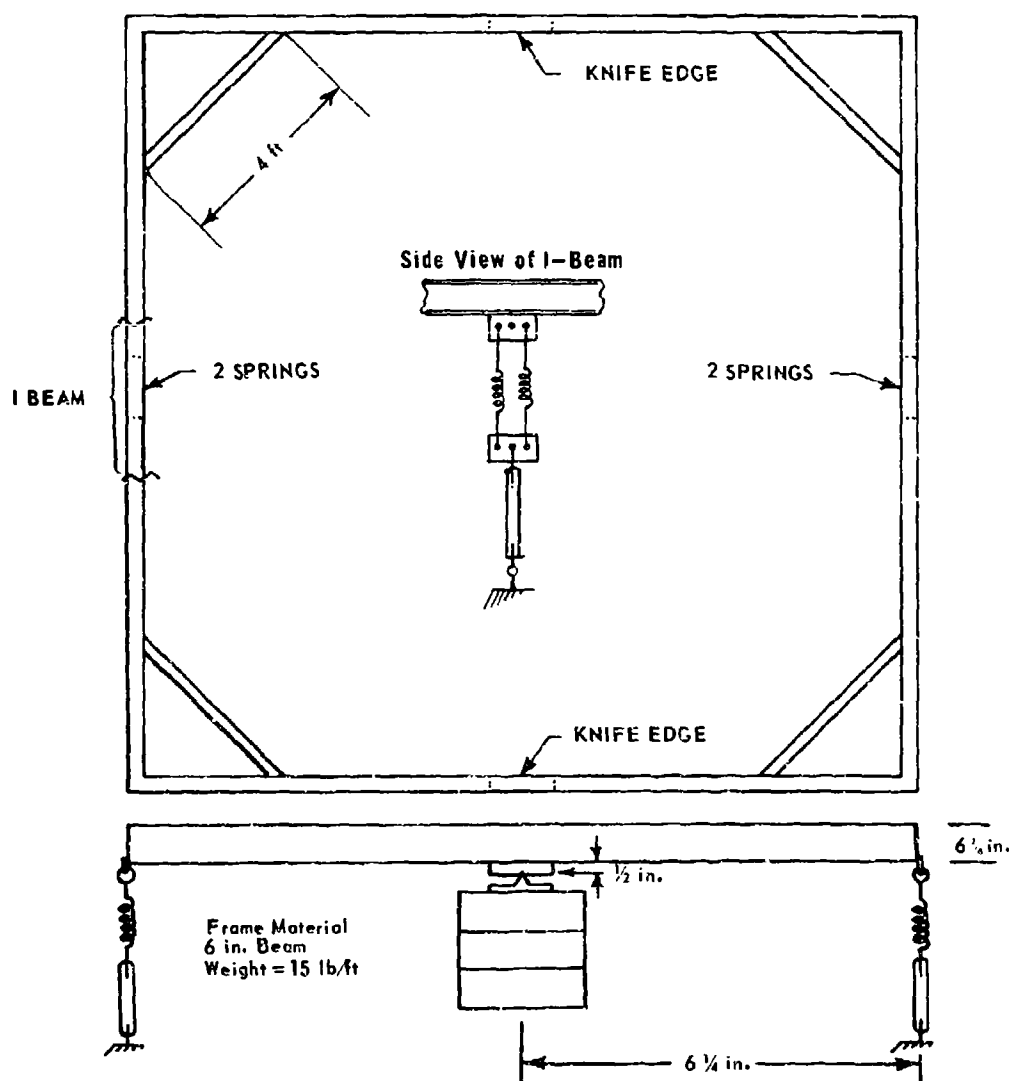


Figure 8 Platform Setup for Measurement of I_x and I_y

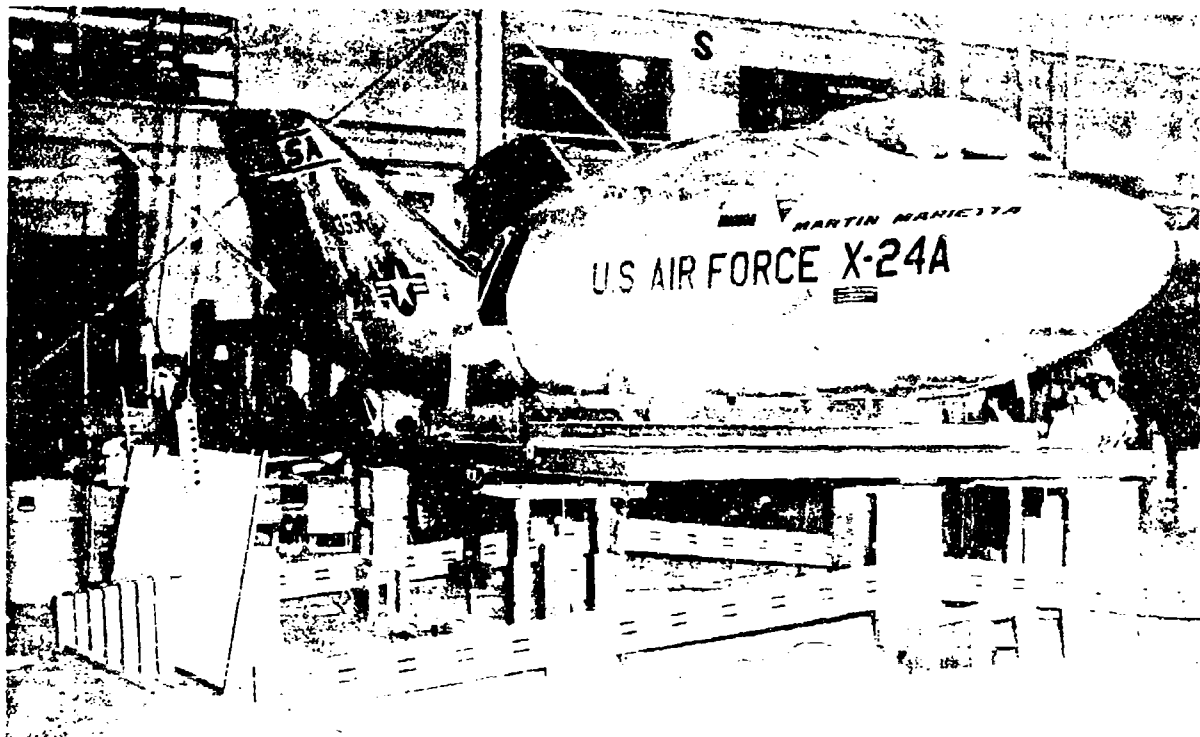


Figure 9 Setup for Measurement of I_y

Figure 9 shows the experimental setup for determining I_y . The entire X-24A and cradle assembly was suspended above the floor on two knife edges, which allowed the vehicle to oscillate about the X axis. Four large springs, two per side, were stretched between the I-beam and the floor with a turnbuckle for pre-loading. Details of the knife edge and springs are shown in figures 10 and 11.

The aircraft and cradle were rotated on the platform in order to use the same knife edges and springs to measure the moment of inertia for pitch. The inertias of the table and table-plus-cradle about the knife edges were determined for both pitch and roll on two separate occasions. Two tests were performed for each inertia measurement. A stopwatch was used to time the period of the oscillations for the first test; the X-24A on-board instrumentation was used for the second. Different springs were used for the two tests. The initial test used two medium springs in order to permit an accurate determination of the period with a watch. The second test used the large springs that were also used to determine the I_z inertia of the X-24A.

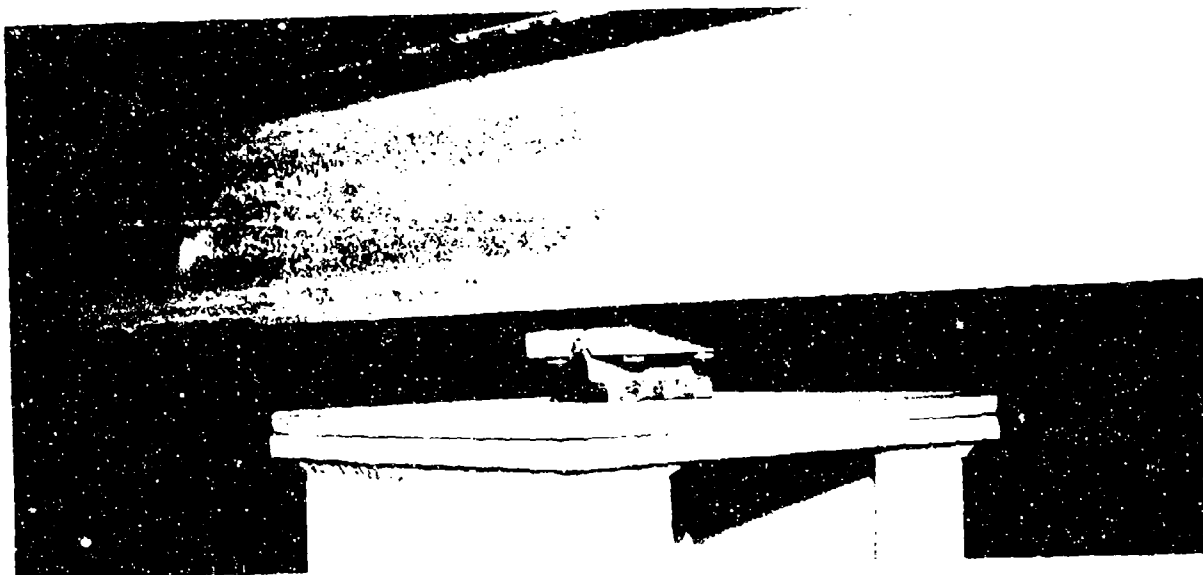


Figure 10 Detail of Knife Edge

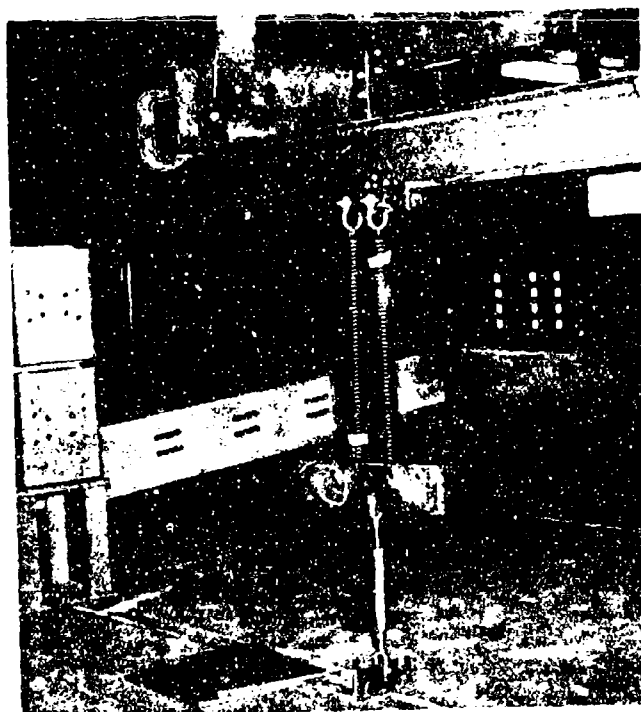


Figure 11 Springs Used for I_y Measurement

COMPUTATION PROCEDURE

I_{table}

The determination of the moment of inertia of the table about the knife edge will be discussed first. The equation for determining the moment of inertia of the table is:

$$I_{\text{table}} = \frac{K_T a^2 - W_T h_T}{\omega^2}$$

Test 1.

Using two medium springs and a stopwatch for timing the period of the oscillation

$$\omega = \frac{20 \text{ cycles}}{23.8 \text{ sec}} = 0.8405 \text{ cycles/sec}$$

$$\omega = 5.278 \text{ rad/sec}$$

$$I_{\text{table}} = \frac{18530 - 282.5}{27.82}$$

$$I_{\text{table}} = 655 \text{ slug-ft}^2$$

Test 2.

Using the X-24A gyros and four large springs

$$K_T = 3061.30 \text{ lb/ft}$$

$$\omega = \frac{20 \text{ cycles}}{9.65 \text{ sec}} = 2.073 \text{ cycles/sec}$$

$$\omega = 13.02 \text{ rad/sec}$$

Substituting constants from table X:

$$I_{\text{table}} = 650 \text{ slug-ft}^2$$

The moments of inertia of the table-plus-cradle and attachments were determined for both pitch and roll from the two methods above. The moment of inertia of the table was the same for both pitch and roll, but the inertia of the table-plus-cradle was not.

$I_{y+c} \text{ (pitch)}$

The determination of the pitch inertia for the table-plus-cradle was as follows:

Test 1.

Using 2 medium springs and a stopwatch for timing the oscillation period and 100 pounds of weight at the 6-foot moment arm of the springs:

$$\omega = \frac{20 \text{ cycles}}{23.1 \text{ sec}} = 0.712 \text{ cycles/sec}$$

$$\omega = 4.47 \text{ rad/sec}$$

$$I_{y_{t+c}} = \frac{K_T a^2 - W_T h_T - W_C h_C}{\omega^2}$$

Substituting constants from table X:

$$I_{y_{t+c}} = 885 \text{ slug-ft}^2 \text{ (pitch)}$$

Test 2.

Using the X-24A gyros and 4 large springs plus 50 pounds of weight at the 6-foot spring moment arm.

$$\omega = 20/11.15 = 1.794 \text{ cycles/sec}$$

$$\omega = 11.27 \text{ rad/sec}$$

Substituting constants from table X:

$$I_{y_{t+c}} = 855 \text{ slug-ft}^2 \text{ (pitch)}$$

This is the inertia used for pitch because of errors in Test 1. (The cradle was shifted 1-3/8 inches from the previous test and only one 50-pound weight at the 6-foot spring moment arm was used instead of 100 pounds as in the previous test.)

The roll inertia of the table plus cradle was determined in the same manner.

$I_{y_{t+c}}$ (roll)

Test 1.

Using two medium springs and a stopwatch to time the period of the oscillation:

$$\omega = 20/28.3 = 0.707 \text{ cycles/sec}$$

$$\omega = 4.44 \text{ rad/sec}$$

Substituting constants from table X:

$$I_{y_{t+c}} = 892 \text{ slug-ft}^2 \text{ (roll)}$$

Test 2.

X-24A instrumentation and four large springs

$$\omega = \frac{20}{11.35} = 1.762 \text{ cycles/sec}$$

$$\omega = 11.08 \text{ rad/sec}$$

$$I_{y_{t+c}} = 893 \text{ slug-ft}^2 \text{ (roll)}$$

To obtain the pitch and roll inertias of the X-24A about the knife edges (KE), the inertias of the combination X-24A, table, and cradle were obtained. The pitch inertia is computed first:

Pitch

$$I_{y_{\text{Combination (KE)}}} = \frac{\overbrace{K_T a^2}^{\text{Spring}} + \overbrace{W_T h_T}^{\text{Table}} - \overbrace{W_C h_C}^{\text{Cradle}} - \overbrace{W_{X-24A} h_{cg}}^{\text{X-24A}}}{\omega^2}$$

where:

$$K_T = 3061.3 \text{ lb/ft (four large springs)}$$

$$W_{X-24A} = 6380 \text{ lb (with dummy pilot & ballast)}$$

$$h_{cg} = \bar{Z} \text{ (WL = 0 to cg)} + \Delta Z \text{ (KE to WL = 0)}$$

$$= 27.09 + 17.516 = 44.606 \text{ in.} = 3.717 \text{ ft}$$

$$\omega = 2.662 \text{ rad/sec}$$

$$I_{y_{\text{Combination (KE)}}} = 12,096 \text{ slug-ft}^2$$

$$I_{y_{X-24A (KE)}} = I_{y_{\text{Combination (KE)}}} - I_{y_{t+c}}$$

$$= 11,241 \text{ slug-ft}^2$$

The moment of inertia about the knife edge was then transferred to the Y-body axis through the cg.

$$I_{y_{\text{body}}} = I_{y_{X-24A (KE)}} - m (h_{cg})^2$$

$$= 11,241 - 2,737$$

$$= 8,504 \text{ slug-ft}^2$$

For the empty aircraft (without pilot, chute and expendable gases):

$$I_y = I_{y_{\text{measured}}} - I_{y_{\text{subtracted}}} + I_{y_{\text{added}}}$$

$$\begin{aligned} I_y &= 8504 - 713.40 + 336.36 \\ &= 8127 \text{ slug-ft}^2 \end{aligned}$$

Roll

A similar procedure was used to determine the roll moment of inertia.

$$I_{x_{\text{combination}}} (\text{KE}) = \frac{\overbrace{K_T a^2}^{\text{Spring}} - \overbrace{W_T h_T}^{\text{Table}} - \overbrace{W_C h_C}^{\text{Cradle}} - \overbrace{W_{X-24} h_{cg}}^{\text{X-24A}}}{\omega^2}$$

where:

$$K_T = 3061.3 \text{ lb/ft (four large springs)}$$

$$\omega = 4.08 \text{ rad/sec}$$

$$I_{x_{\text{combination}}} (\text{KE}) = 5152$$

$$I_{x_{X-24A}} (\text{KE}) = I_{x_{\text{combination}}} (\text{KE}) - I_{x_{t+C}}$$

$$= 5152 - 893$$

$$= 4259 \text{ slug-ft}^2$$

$$I_{x_{\text{Body}}} = I_{x_{X-24A}} (\text{KE}) - m(h_{cg})^2$$

$$I_{x_{\text{Body}}} = 4,259 - 2,736 = 1,521 \text{ slug-ft}^2$$

For the empty aircraft (without pilot, chute and expendable gases):

$$I_x = I_{x_{\text{measured}}} - I_{x_{\text{subtracted}}} + I_{x_{\text{added}}}$$

$$I_x = 1521 - 29.85 + 83.65$$

$$I_x = 1565 \text{ slug-ft}^2$$

MOMENT OF INERTIA SUMMARY

Tables XI and XII present the measured X-24A moments of inertia.

For comparison purposes, the X-24A weight and moments of inertia at launch computed by the Martin Company prior to vehicle delivery are shown below. Also shown are actual first flight values. The data shows that the actual vehicle weight and moments of inertia are larger than computed by the contractor. It must be noted, however, that there were numerous aircraft weight changes between the Martin Company determination and the measurement described in this paper.

Table XI

X-24A MOMENT OF INERTIA SUMMARY

Configuration	Weight (lb)	I_{xx}	I_{yy} (slug-ft ²)	I_{zz}	I_{xz}
At inertia measurement ¹	6,380	1,521	8,504	9,440	0.0
Empty aircraft - first flight ¹	5,917	1,565	8,127	8,912	79.76

NOTE:

1. The values of I_x and I_y that were used as a baseline during the flight program were lowered by 25 slug-ft² due to a computational error discovered late in the test program. All calculations of flight inertias are based on empty aircraft values of $I_x = 1540$ and $I_y = 8102$ slug-ft².

Table XII

X-24A INERTIA COMPARISON WITH PREDICTIONS

Configuration ¹	Weight (lb)	I_{xx}	I_{yy} (slug-ft ²)	I_{zz}	I_{xz}
Martin Co. estimate	6,006.69	1,246.9	7,180.8	7,787.9	144.2
Actual first flight ¹	6,362	1,543.9	8,537.9	9,345.5	39.9
Difference with actual slug-ft ² (pct)	356 (+5.6)	297 (+19.3)	1,357 (+15.9)	1,558 (+17.7)	-104.0

NOTE:

1. Full aircraft ready for launch for a glide flight.

DETERMINATION OF INFLIGHT WEIGHT AND BALANCE

TEST PROCEDURE

Since the first flight of the X-24A, all configuration changes which affected the weight of the airplane were recorded and used to update the mass data of the basic airplane. Prior to each flight, the launch and landing cg's were predicted. The first nine flights and the twenty-second flight of the X-24A were glide flights with no XLR-11 rocket propellants on board. Required data for these flights were the dry weight (basic airplane), launch weight with pilot and expendable gasses, and landing weight which varied only if the hydrogen peroxide landing rockets were used. This information is shown in table XII. The remaining flights were powered, and the mass data were a function of the propellant flow rates and propellant angles. A computer program was developed to compute the mass data at discrete times throughout the flight. Time histories of cg's and moments of inertia for flights X-10-15 through X-28-34 (excluding X-22-27) are shown in appendix VI.

COMPUTER PROGRAM

An X-24A mass data program was written for the IBM 7094 computer. The program listout is shown in appendix V. Subroutines are used to compute configuration changes and mass changes due to peroxide flow, LOX prime, water-alcohol prime, propellant flow for each rocket chamber, and propellant jettison. Also included are subroutines to compute the effects of propellant angles which result from aircraft accelerations. The individual computer subroutines are described below in their order of use in powered flight analyses.

Addition and Subtraction Subroutines

These two subroutines, labeled ADDAT and MINUS, respectively, are used to add or delete new mass items from the aircraft and calculate new cg's and moments of inertia. The subroutines accomplished the bookkeeping task of accounting for weight changes to the aircraft between flights. Updated weights were checked by comparing them to periodic weighings at the AFFTC weight and balance facility. Any differences were noted and the new measured weight and horizontal cg were used as a baseline for the ensuing flights. A comparison of predicted weights and actual weighings is shown in appendix VII.

After updating the new weight of the empty airplane, the full, captive flight mass properties were determined by adding the point masses of the pilot and chute, expendable gases (cabin air, helium, emergency helium, hydrogen peroxide), and, if required, the propellants (liquid oxygen and water-alcohol). Point mass amounts of LOX and water alcohol used were 2,760 pounds and 2,510 pounds, respectively. These values were measured by ground fill tests. The volume of LOX on board during actual flights probably varied somewhat from the ground test measurement due to differences in LOX density at varying temperatures. Since LOX temperature was not monitored in flight, no correction was possible. The fully serviced values of 19.7 pounds cabin air, 200.0 pounds hydrogen peroxide, 11.7 pounds helium, and 2.1 pounds emergency helium were handbook values (reference 10).

Other prelaunch losses of cabin air and helium were calculated with the subtraction subroutine. These values were average values on typical prelaunch flight times and leakage rates and were the same for all flights. The cabin air loss was 1.2 pounds, and the helium loss was 2.0 pounds. No accurate method was found to determine the LOX boil-off at altitude between LOX top-off and tank pressurization. This time period was normally less than 30 seconds so this loss was neglected.

Prime Subroutines

Prelaunch mass losses due to rocket engine prime were calculated in the following subroutines: PEROX, LOXPRIM, WALPRIM. The start of prime was determined by the drop in temperature of the LOX prime line measured by a thermocouple on the line itself. Approximately 20 seconds later a second, smaller slope change with time of LOX prime line temperature was the indication of the change from gaseous to liquid LOX prime. The gaseous LOX prime rate of 0.313 pound per second, the liquid LOX prime rate of 3.84 pounds per second, and the water-alcohol prime rate of 0.02 pound/second were measured on a ground test engine run. The assumption was made that the propellant flow rates during the engine igniter test were the same as the prime flow rates for this short period of time (approximately one second).

Propellant Angle Subroutines

For a partial load of fuel and oxidizer the vehicle cg was a strong function of the location of the fluid in the individual tanks. The surface of the fluid would be perpendicular to the total resultant force vector on the aircraft. The angle between the resultant force vector and the fuselage reference line was determined by resolving the normal and longitudinal accelerometer readings at any instant in time. This angle is called the propellant angle (θ_p) and is defined as

$$\theta_p = \sin^{-1} \left(\frac{X_B}{\sqrt{Z_B^2 + X_B^2}} \right) = \sin^{-1} \left(\frac{X_B}{\sqrt{R_F^2}} \right)$$

where X_B and Z_B are the X and Z body axis forces, respectively (or accelerometer readings).

Estimates for the location of the cg of various amounts of trapped propellants were made using trapezoidal approximation for the tank shapes. These computations were made in increments of 1/8 of the total propellant load over a range of propellant angles of +90 degrees. These data are shown in table I of appendix VIII. The cg locations of LOX and water-alcohol are used along with flight measurements of propellant angle and propellants remaining to calculate the actual cg and moments of inertia versus time for each powered flight. To obtain the maximum cg travel with propellant angle for purposes of simulation and flight planning, the matrix of all combinations of weights and fuel angles was run through the X-24A weight and balance computer program to compute total aircraft weight and cg for each condition. Estimates of the ratio of LOX and water-alcohol on board at any particular time (or gross weight) were based on LOX top-off and boil-off estimates, prime estimates, and engine specification values for oxygen/fuel ratio. The resulting data produced curves such as the peanut-shaped curve for horizontal cg shown in figure 12. The

changes in all other mass properties as functions of propellant angle were also calculated.

To compensate for propellant angle changes while the engine was running, a new weight and cg were computed every 10 seconds during the burn using the LOX, WAL, and PEROX subroutines. Starting at the time of ignition, LOX, water-alcohol, and hydrogen peroxide are subtracted as a function of time of burn, flow rate, propellant angle, and number of chambers burning. The sudden propellant angle change due to acceleration changes at engine start and shutdown was accounted for with the LOXDEL and WALDEL subroutines which computed a cg change without any change in mass.

One of the weak points of determining the mass properties during powered flight was the lack of knowledge of actual engine propellant flow rates. Initially, specification values of flow rates were used in the calculations. Later in the powered flight program, attempts to determine flow rates from ground tests were not totally successful. (Obviously, a better way to determine propellant flow rates would be through the use of flowmeters installed in the propellant lines.) Representative LOX and water-alcohol flow rates used during the program were 4.75 and 4.40 pounds per second per chamber, respectively.

Jettison Subroutine

If jettison occurred during a flight, LOX and water-alcohol were subtracted separately at rates of 73.0 and 63.0 pounds per second, respectively. These values were determined by test jettisons on the ground which were timed with a stopwatch. Decay of tank pressures was used to determine the time of jettison of the propellants.

Hydrogen Peroxide Subroutine

Hydrogen peroxide is subtracted at different flow rates, depending on whether it is used for engine prime, jettison, landing rockets, or any combination of engine chambers. If landing rockets are used, the flow rate is 7.1 pounds per second, which was determined from an engine ground run early in the program. Flow rates for the XLR-11 rocket engine are as follows (reference 10):

Prime - 0.013 lb/sec	3 Chambers - 0.45 lb/sec
1 Chamber - 0.31 lb/sec	4 Chambers - 0.51 lb/sec
2 Chambers - 0.32 lb/sec	Jettison - 5.26 lb/sec

Landing Gear Effects

The effect of lowering the landing gear was calculated for six separate flights based on the individual landing weight for each flight and design estimates of gear weights and cg locations. The effects were averaged for the six flights and the average incremental values were applied to all of the other flights. The calculated average shifts were: -2.002 inches for \bar{X} , 0.0 for \bar{Y} , -2.461 inches for \bar{Z} , +158.17 slug-ft² for I_{xx} , +107.63 slug-ft² for I_{yy} , -50.80 slug-ft² for I_{zz} , and 12.905 slug-ft² for I_{xz} .

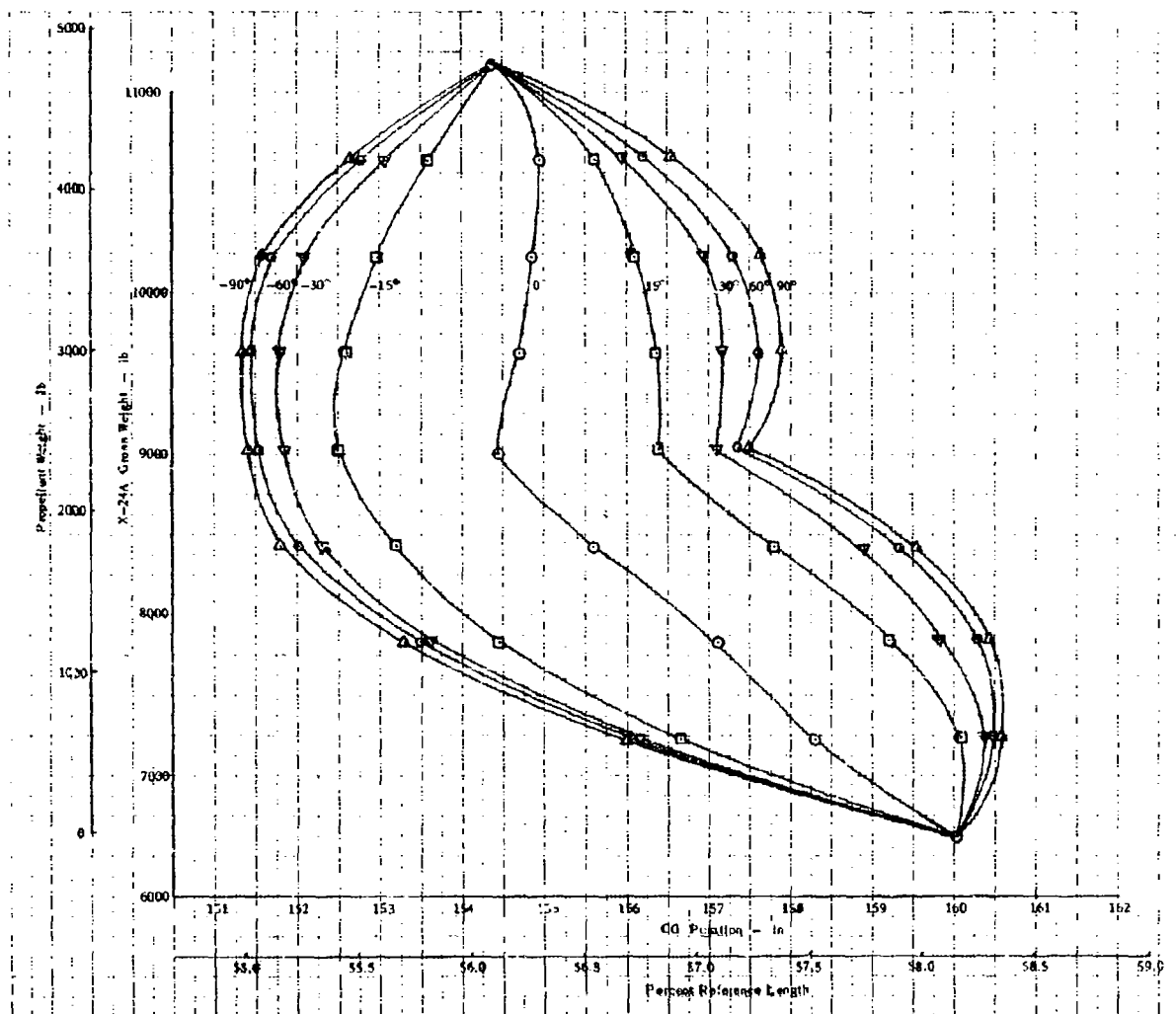


Figure 12 X-24A Variation in cg Position with Weight and Propellant Angle

Table XIII

X-24A MASS PROPERTIES DATA

(Flights X-1-2 through X-f-14 and Flight X-22-27)

Flight	Weight (lb)	\bar{X} (in.)	\bar{X} (pct)	\bar{Y} (in.)	\bar{Z} (in.)	I_x (slug-ft ²)	I_y (slug-ft ²)	I_z (slug-ft ²)	I_{xz} (slug-ft ²)
X-1-2 (empty)	5,917.5	162.79	58.98	-0.08	27.03	1840.0	8102.0	8912.0	73.0
X-1-2 (launch)	6,352.5	160.72	58.23	-0.18	27.18	1543.9	8537.9	9345.5	39.9
X-1-2 (landing)	*6,160.8	159.68	57.86	-0.07	27.29	1542.8	8490.1	9297.7	45.1
X-2-3 (empty)	5,917.5	162.79	58.98	-0.08	27.03	1540.0	8102.0	8912.0	73.0
X-2-3 (launch)	6,352.5	160.72	58.23	-0.18	27.18	1543.9	8537.9	9345.5	39.9
X-2-3 (landing)	*6,263.0	160.24	58.06	-0.13	27.23	1543.4	8515.5	9323.6	42.3
X-3-5 (empty)	5,927.1	162.72	58.95	-0.03	25.96	1541.8	8080.4	8891.4	70.1
X-3-5 (launch)	6,362.0	160.65	58.21	-0.14	27.11	1545.7	8515.9	9324.5	36.8
X-3-5 (landing)	*6,170.3	159.61	57.83	-0.03	27.22	1544.6	8467.9	9297.5	41.9
X-4-7 (empty)	6,000.0	163.46	59.22	-0.05	25.92	1548.8	8159.6	8970.9	75.4
X-4-7 (launch)	6,434.9	161.36	58.46	-0.15	27.07	1552.7	8599.1	9408.0	41.8
X-5-8 (empty)	5,860.3	161.05	58.35	-0.04	27.09	1471.9	7841.9	8578.3	97.0
X-5-8 (launch)	6,295.2	159.08	57.64	-0.14	27.23	1475.8	8268.7	9003.7	64.4
X-6-10 (empty)	5,864.1	160.98	58.33	-0.02	27.05	1473.1	7846.9	8584.6	98.5
X-6-10 (launch)	6,299.0	159.01	57.61	-0.12	27.21	1476.9	8273.4	9008.7	65.7
X-7-11 (empty)	5,863.9	160.99	58.33	-0.04	27.06	1472.8	7844.9	8581.7	98.2
X-7-11 (launch)	6,298.8	159.03	57.62	-0.15	27.21	1476.7	8271.5	9005.9	65.5
X-8-12 (empty)	5,863.9	160.99	58.33	-0.04	27.06	1472.8	7844.9	8581.7	98.2
X-8-12 (launch)	6,298.8	159.03	57.62	-0.15	27.21	1476.7	8271.5	9305.9	65.5
X-9-14 (empty)	6,018.7	160.29	58.08	-0.05	26.48	1488.3	7872.2	8595.5	107.6
X-9-14 (launch)	6,463.4	158.30	57.35	-0.13	26.71	1491.6	8313.7	9034.4	72.0
X-22-27 (empty)	5,893.0	161.32	58.45	-0.06	26.74	1431.3	7697.2	8429.3	49.5
X-22-27 (launch)	6,336.0	159.20	57.68	-0.16	26.92	1435.4	8144.5	8873.9	14.6

*Hydrogen peroxide landing rockets were used only on the first three flights. For all other glide flights, the launch and landing weights are identical.

APPENDIX I **WEIGHT AND cg MEASUREMENTS**

AIRPLANE WEIGHING RECORD					
DATE WEIGHED		MODEL	SERIAL NUMBER		
25 Nov 68		X-24	551		
PLACE WEIGHED		WEIGHING PERSONNEL			
EAFB, Calif		R. Anderson and G. Hellwig			
REACTION (Wheels, jacks, etc.)	SCALE READING	TARE	NET WEIGHT	ARM	MOMENT
LEFT MAIN					
RIGHT MAIN					
SUB-TOTAL (Both Mains)			5360	179.58	962,549
NOSE			1020	62.28	63,526
TOTAL (As Weighed)			6380	160.83	1,026,075

MEASUREMENTS

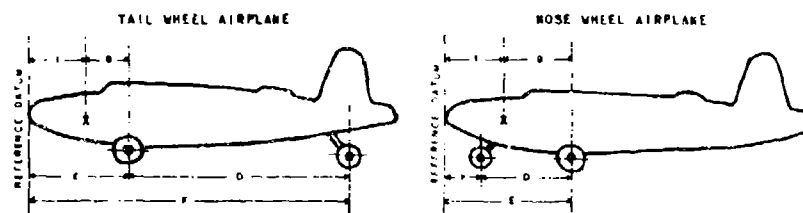
R = 4.58 the distance from the jig point, to the center line of the main reactions. Obtain by measurement.

I = 175.00 the distance from the reference datum to the jig point of the airplane, from which a plumb bob can be dropped to the ground. Obtain from the airplane diagram in Chart E.

E = 179.58 ^{1/2} the distance from the reference datum to the center line of the main reactions.
 $E = I + B$
 $E = I - R$ (if the jig point is aft of the center line of the main reactions.)

D = 117.30 the wheel base (or the distance between fore and aft reactions.) Obtain by measurement.

F = 62.28 ^{1/2} the distance from the reference datum to the center line of the nose or tail reaction.
 $F = E - D$ (For nose wheel type aircraft) Grnd. Att. = 2°52' Nose up
 $F = E + D$ (For tail wheel type aircraft)



DIAGRAMS FOR MEASURING VARIOUS TYPES OF AIRPLANES TO DETERMINE ARM OF SUPPORT POINTS.

^{1/2} Check dimensions E and F against approximate dimensions listed on Chart E.

Figure 1 X-24A Weight and Balance Records

AIRPLANE WEIGHING RECORD					FOR USE IN T.O. 1-12-40 & AM 01-12-40	
DATE WEIGHED 25 Nov 68		MODEL X-24		SERIAL NUMBER 551		
PLACE WEIGHED EAFB, Calif		WEIGHING PERSONNEL R. Anderson and G. Hellwig				
REACTION (Wheels, jacks, etc.)	SCALE READING	TARE	NET WEIGHT	ARM	MOMENT	
LEFT MAIN						
RIGHT MAIN						
SUB-TOTAL (Both Main)			5210	180.88	942,385	
NOSE OR TAIL			1170	63.68	74,506	
TOTAL (As Weighed)			638	159.39	1,016,891	

MEASUREMENTS

Long. C.G. after adding ballast (100 lb)

B = 5.88 the distance from the jig point, to the center line of the main reactions. Obtain by measurement.

I = 175.00 the distance from the reference datum to the jig point of the airplane, from which a plumb bob can be dropped to the ground. Obtain from the airplane diagram in Chart E.

E = 180.88 1/ the distance from the reference datum to the center line of the main reactions.
 $E = I + B$
 $E = I - B$ (If the jig point is aft of the center line of the main reactions.)

D = 117.20 the wheel base (or the distance between fore and aft reactions.) Obtain by measurement.

F = 63.68 1/ the distance from the reference datum to the center line of the nose or tail reaction.
 $F = F - D$ (For nose wheel type aircraft)
 $F = E + D$ (For tail wheel type aircraft)

Aft J.P. to MR = 0.49'
 NR to MR = 9.767'

TAIL WHEEL AIRPLANE

NOSE WHEEL AIRPLANE

DIAGRAMS FOR MEASURING VARIOUS TYPES OF AIRPLANES TO DETERMINE ARM OF SUPPORT POINTS.

1/ Check dimensions B and F against approximate dimensions listed on Chart E.

Figure 2 X-24A Weight and Balance Records

AIRPLANE WEIGHING RECORD					
DATE WEIGHED 25 Nov 68		MODEL X-24		SERIAL NUMBER 551	
PLACE WEIGHED EAFB, Calif		WEIGHING PERSONNEL R. Anderson and G. Hellwig			
REACTION (Wheels, jacks, etc.)	SCALE READING	TARE	NET WEIGHT	ARM	MOMENT
LEFT MAIN					
RIGHT MAIN					
Left Main	2660	- 18	2642	E	
Right Main	2660	- 18	2642	F	
NOSE			1096	0.00	
TOTAL (As Weighed)			6380	0.00	

MEASUREMENTS

Lateral C.G.

B = _____ the distance from the jig point, to the center line of the main reactions. Obtain by measurement.

I = _____ the distance from the reference datum to the jig point of the airplane, from which a plumb bob can be dropped to the ground. Obtain from the airplane diagram in Chart E.

E = _____ // the distance from the reference datum to the center line of the main reactions.
 $E = I + B$
 $E = I - B$ (If the jig point is aft of the center line of the main reactions.)

D = _____ the wheel base (or the distance between fore and aft reactions.) Obtain by measurement.

F = _____ // the distance from the reference datum to the center line of the nose or tail reaction.
 $F = F - D$ (For nose wheel type aircraft)
 $F = E + D$ (For tail wheel type aircraft)

LM

□

RM

□

C

↓

TAIL WHEEL AIRPLANE

NOSE WHEEL AIRPLANE

DIAGRAMS FOR MEASURING VARIOUS TYPES OF AIRPLANES TO DETERMINE ARM OF SUPPORT POINTS.

// Check dimensions E and F against approximate dimensions listed on Chart E.

Figure 3 X-24A Weight and Balance Records

APPENDIX II SPRING CALIBRATIONS

Spring Number 1 (Small)

$$K_1 = 120.8 \text{ lbs/ft}$$

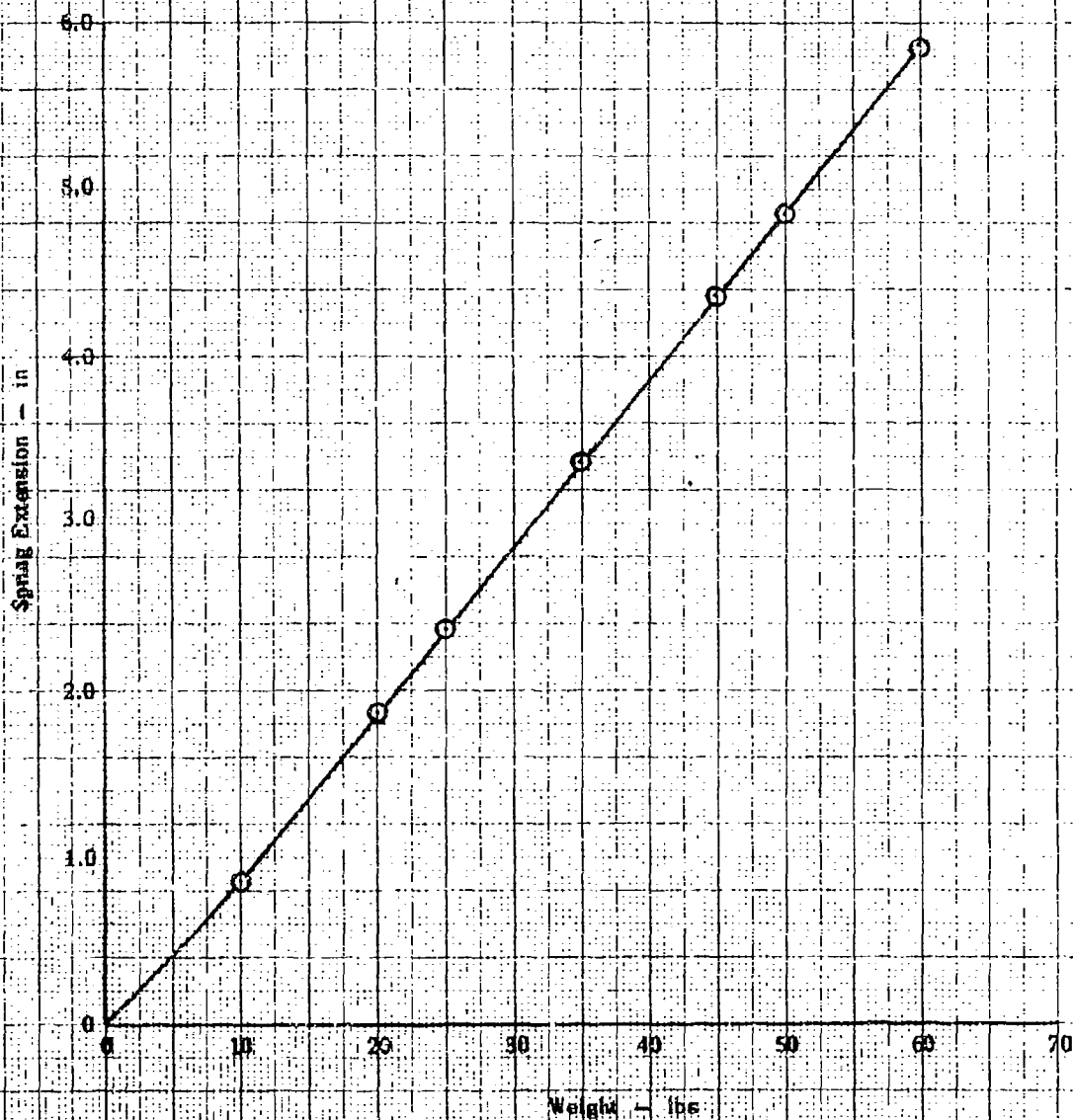


Figure 1 Spring Calibrations

SPRING Number 2 (Small)
 $R_2 = 118.9 \text{ lbs/in}$

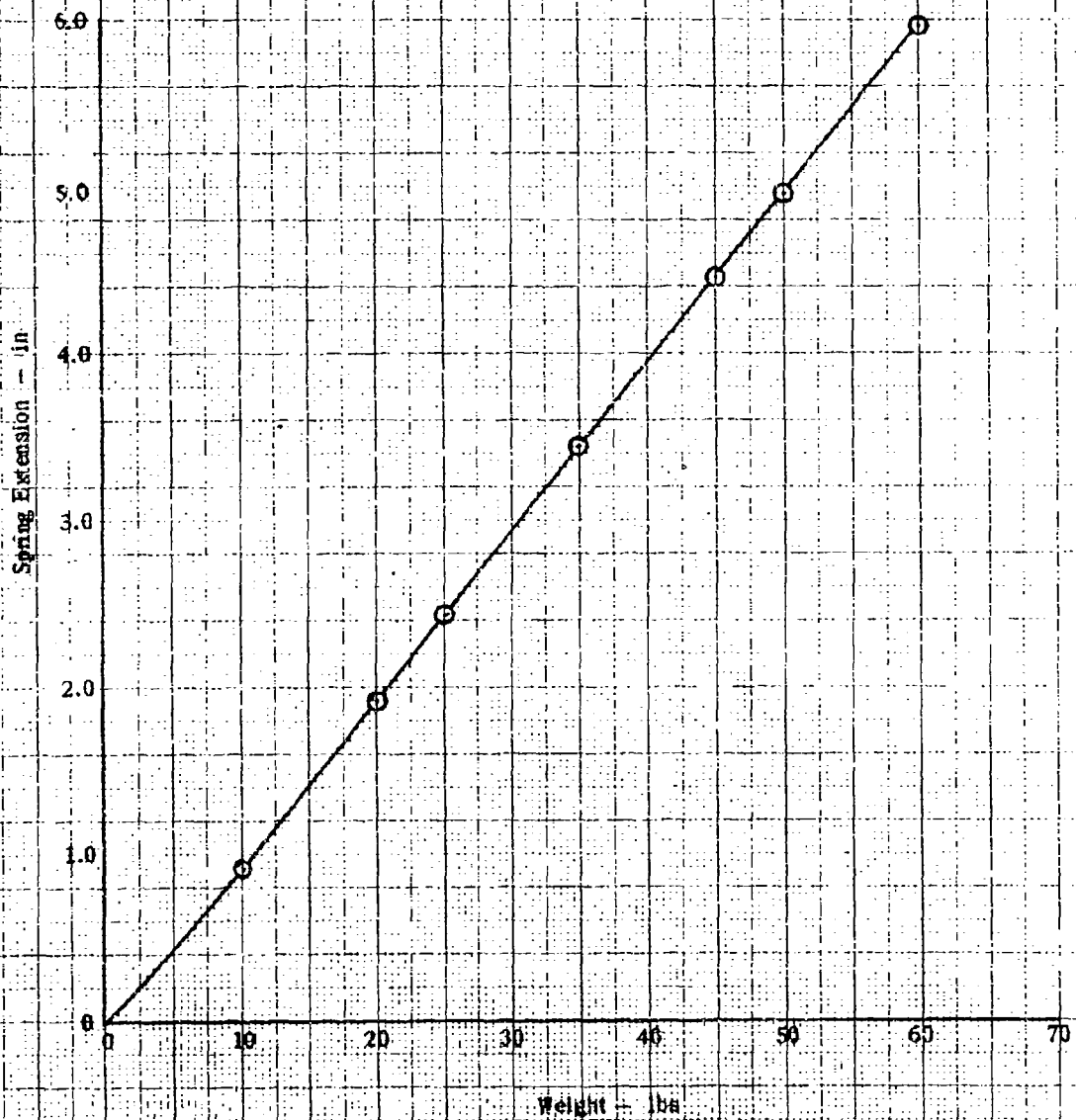
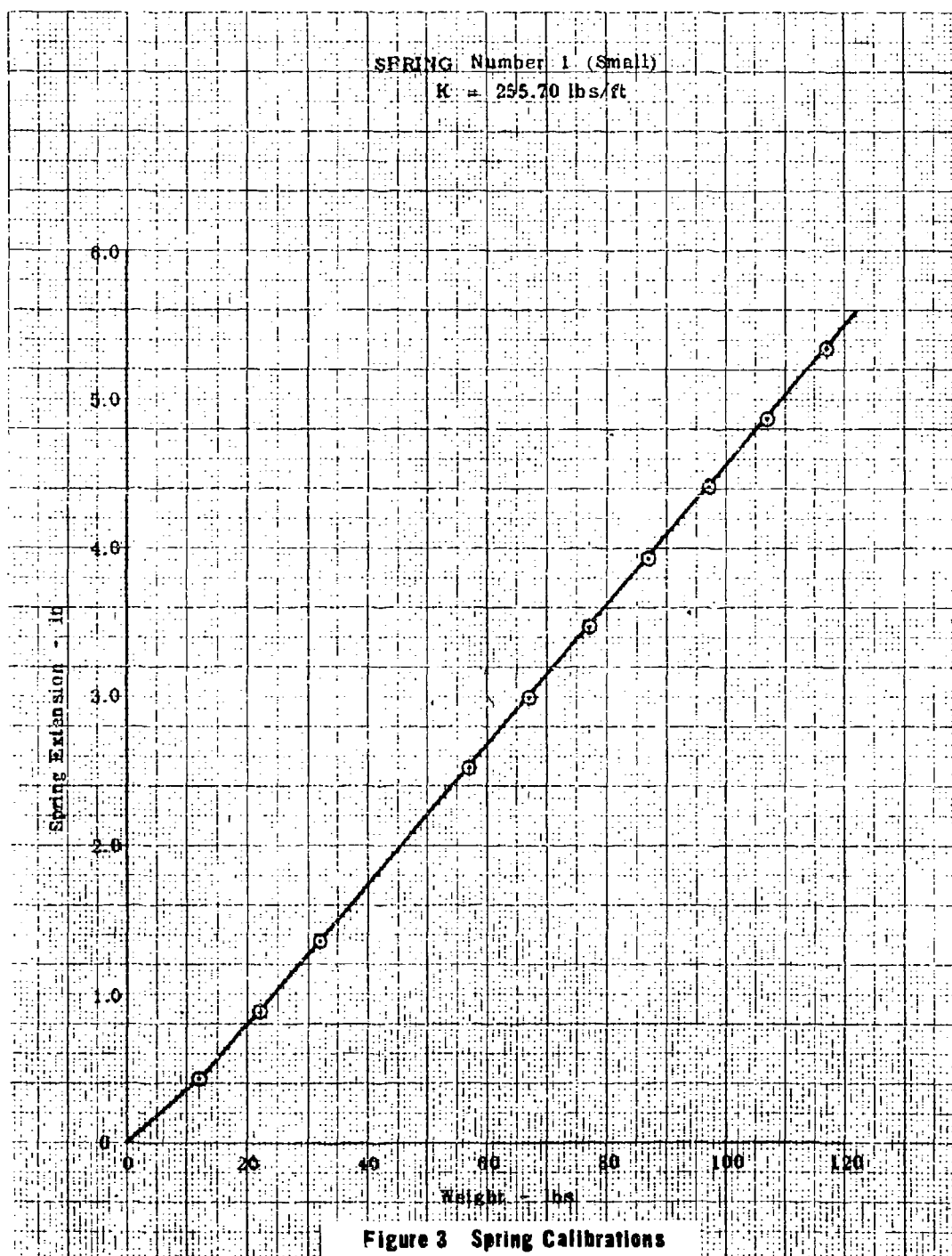


Figure 2 Spring Calibrations



SPRING Number 2 (Small)

$K = 259.70 \text{ lbs/ft}$

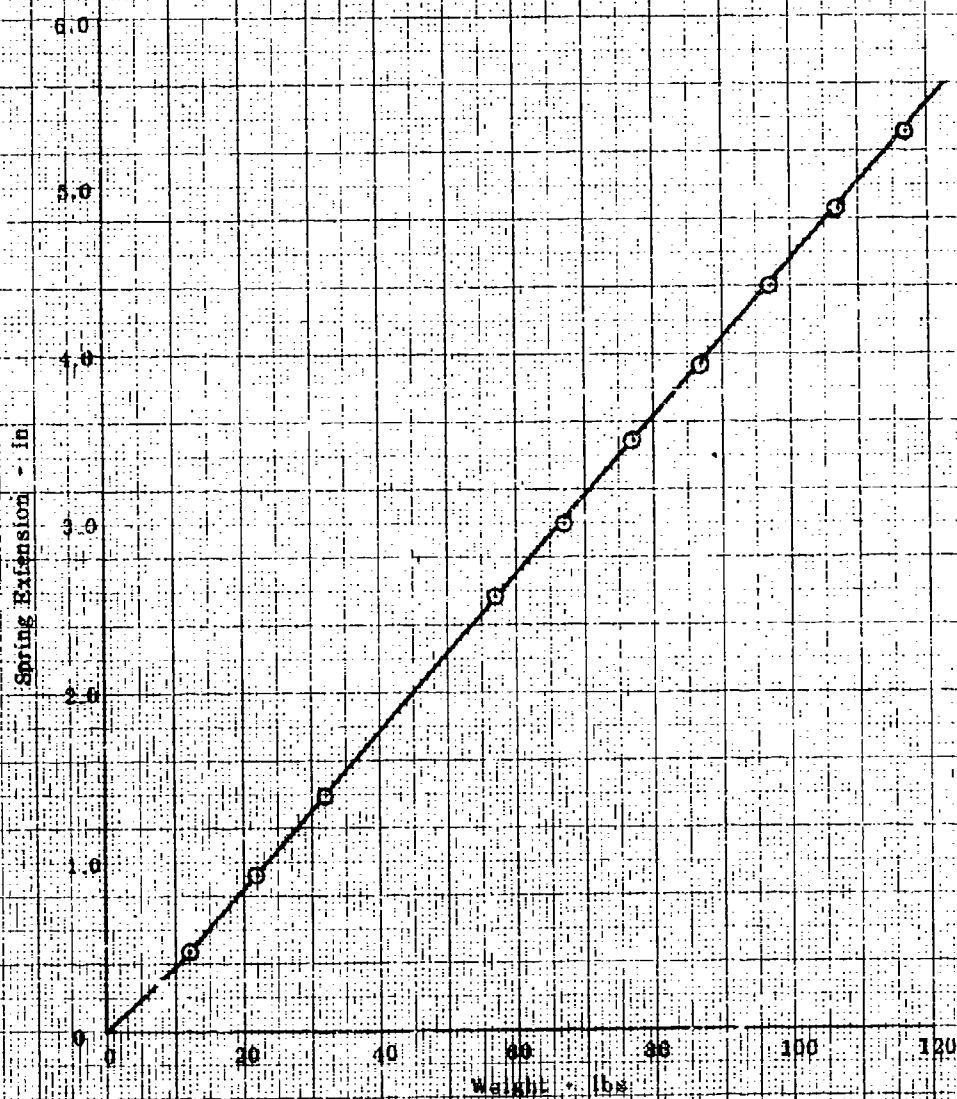


Figure 4 Spring Calibrations

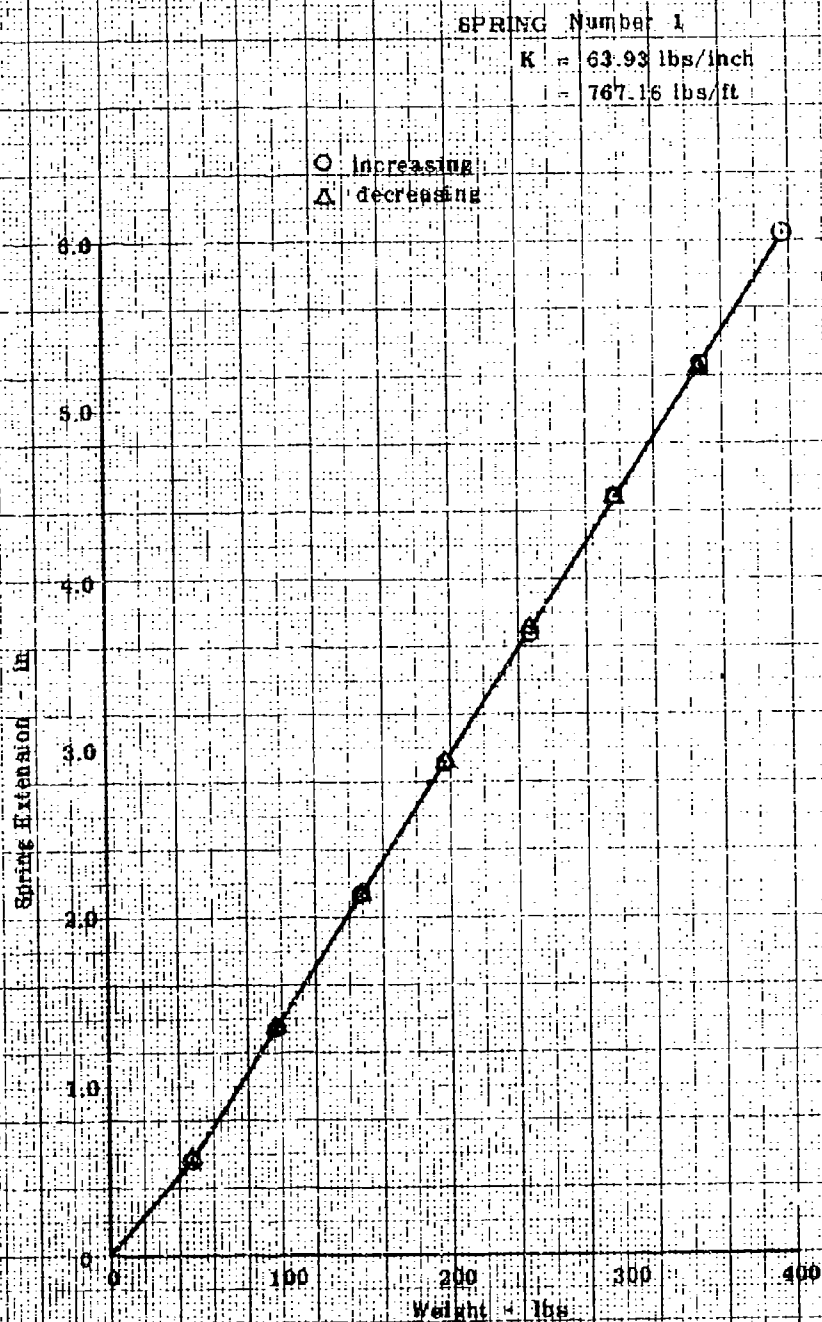
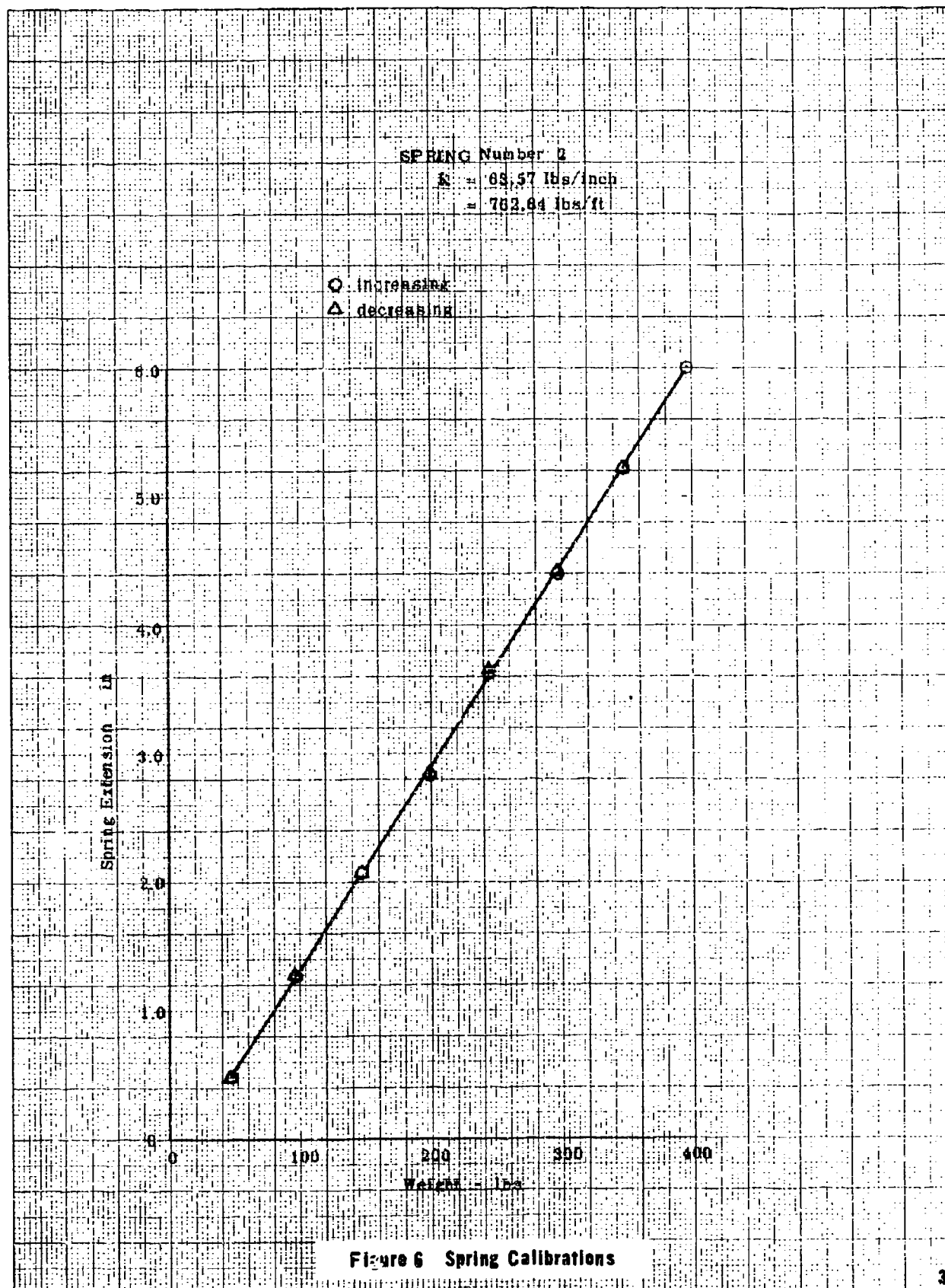


Figure 5 Spring Calibrations



SPRING Number 31
K = 63.73 lbs/inch
= 764.48 lbs/ft

○ increasing
△ decreasing

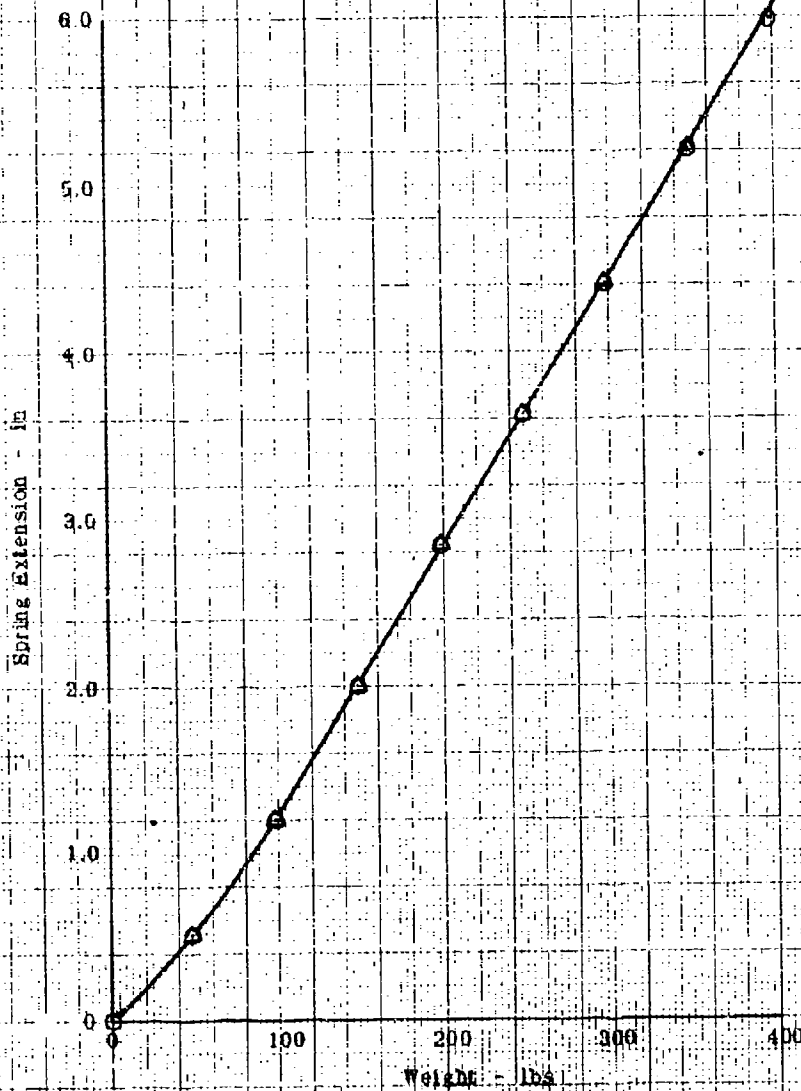
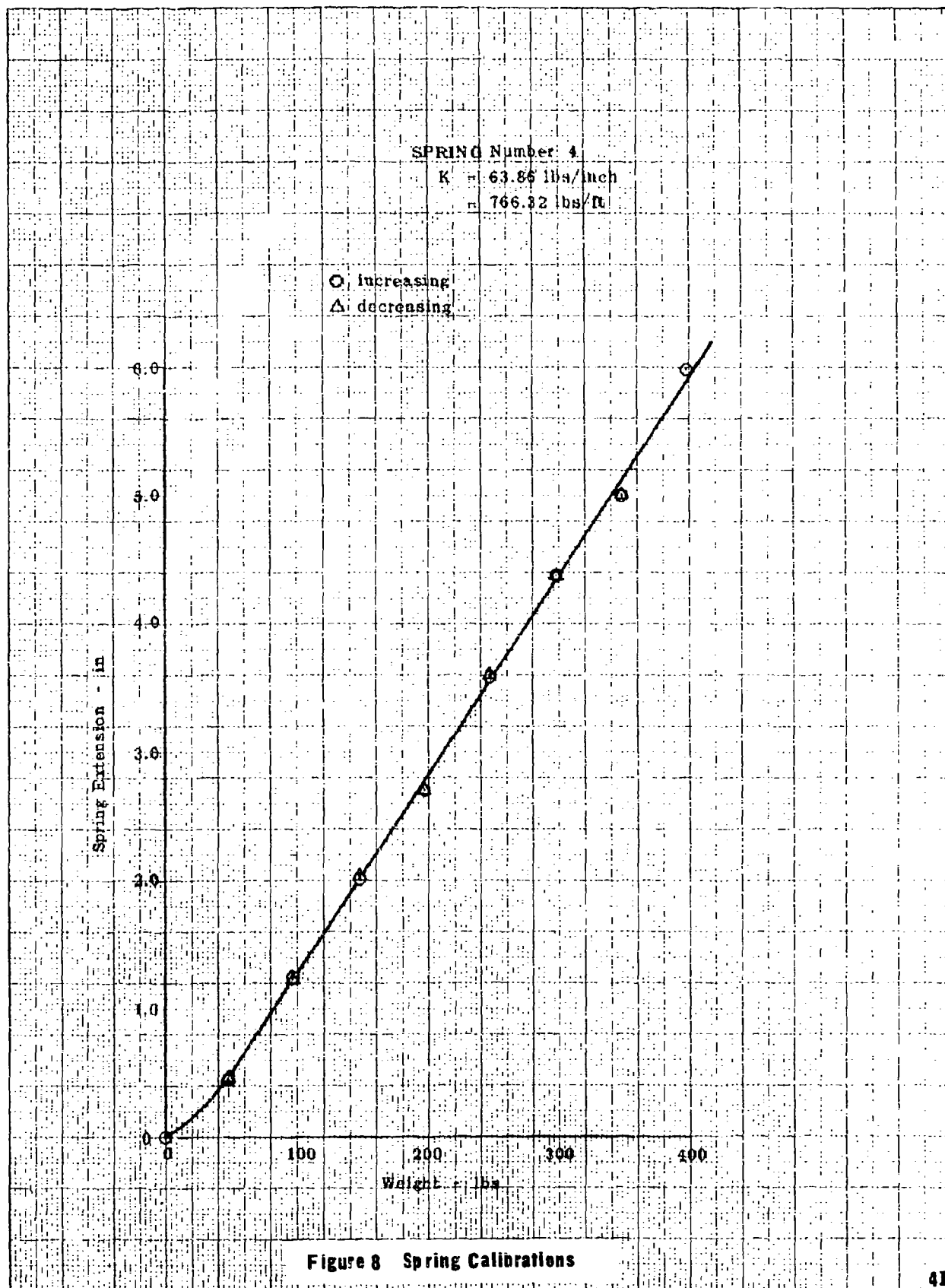


Figure 7 Spring Calibrations



APPENDIX III

ITEMIZED LISTING OF COMPONENTS ADDED AND SUBTRACTED

Table I computes the inertia increments of all items added and subtracted from the X-24A between the inertia measurement and the first flight. Columns 17, 18, and 20 show the inertias for each axis. Table III summarizes the inertias for items removed from the aircraft prior to the first flight. Tables IV and V present the same summary for the vertical moments used to determine the vertical cg.

Table I
MOMENT OF INERTIA CALCULATIONS

ITEM	(1) Weight	(2) Station	(3) Water Line	(4) Bulkhead Line	(5) Vertical Pivot Arm B/L to (3)	(6) Moment about Pivot Weight x (5)	(7) X	(8) Y	(9) Z	(10) X^2	(11) Y^2	(12) Z^2	(13) $X^2 + Y^2 + Z^2$	(14) $X^2 + Y^2$	(15) $X^2 + Z^2$	(16) $Y^2 + Z^2$	(17) I_x	(18) I_y	(19) I_z	(20) $I_{x^2+y^2+z^2}$	
Ballast Weight	14.375	235.75	9.5	12.0 L	77.66	-1044.49	12.49	-32.0	15.36	38.3	7.11	1.53	8.64	39.83	45.41	3.86	17.78	20.27	-7.57	-3.42	
	28.50	237.25	9.0	20.0 L	73.16	-2085.06	-75.79	-20.0	15.86	39.89	4.77	1.64	4.31	41.51	17.66	7.81	36.75	37.76	-8.084	-7.16	
	22.156	231.75	9.0	20.25 L	73.16	-1620.93	-70.29	-20.25	15.86	34.31	2.85	1.64	4.49	39.95	37.16	3.09	27.45	25.57	-7.53	-5.16	
	29.0	256.0	9.5	9.0 L	72.66	-2107.14	-74.54	-9.0	15.36	33.56	.563	1.53	2.09	40.11	39.14	1.88	35.12	35.25	-7.69	-6.93	
	4.5	238.25	9.5	9.0 L	72.66	-326.97	-77.04	-9.0	15.36	41.29	.563	1.53	2.09	42.75	41.78	.29	5.97	5.84	-7.75	-1.10	
	25.375	238.75	9.0	3.0 L	73.16	-2075.92	-77.29	-3.0	15.86	41.48	.063	1.64	1.70	43.12	41.54	1.50	38.0	36.61	-4.74	-7.26	
	14.375	232.50	11.5	19.25 R	70.66	-1015.74	-71.04	19.25	13.32	35.05	2.57	1.14	3.71	36.19	37.62	1.66	16.15	16.74	-6.32	-2.82	
	28.50	237.50	10.0	20.25 R	72.16	-2056.56	-75.04	20.25	14.86	40.15	2.95	1.43	4.28	41.58	43.0	3.79	35.80	36.05	58.1	-5.71	
	14.375	232.0	10.25	19.25 R	71.91	-1033.71	-70.50	19.25	14.61	34.57	2.57	1.35	3.95	35.90	34.59	1.76	16.03	15.22	-5.91	-3.09	
	14.50	231.75	9.5	19.25 R	72.66	-1053.57	-70.29	19.25	15.35	34.71	2.57	1.53	4.10	35.81	36.88	1.85	15.14	16.61	-7.75	-3.26	
Ballast Weight Dummy Pilot	29.0	231.125	9.75	7.0 R	72.41	-2099.89	-69.67	7.0	15.11	33.71	.34	1.39	1.73	35.16	34.05	1.56	31.61	30.67	-7.07	-6.27	
	78.5	232.25	9.75	7.75 R	72.41	-2063.69	-70.79	7.75	15.11	34.80	.42	1.39	1.85	36.19	35.72	1.64	32.02	31.17	-7.18	-6.75	
	25.0	271.25	28.87	14.75 L	53.29	-1332.25	-109.79	14.75	-5.01	83.70	1.51	.14	1.65	83.84	85.21	1.28	65.09	66.16	3.44	2.57	
	25.0	253.0	28.87	9.25 L	53.29	-1332.25	-91.54	-9.25	-5.01	58.19	0.59	.14	.73	58.33	59.78	.57	45.44	45.64	2.87	7.23	
	50.0	253.0	29.37	14.375 R	52.97	-2648.50	-51.54	14.38	-5.50	58.19	1.44	.17	1.61	58.36	59.63	2.50	90.62	92.59	3.16	4.94	
	217.0	70.0	34.0	0.0	48.16	-10450.72	91.46	0.0	-10.13	58.90	0.0	.64	64	59.54	58.90	4.31	401.25	396.90	—	—	
	7.0	120.0	40.50	41.39	-41.66	-291.62	41.45	-41.38	-16.64	11.92	11.89	1.81	13.7	13.8	23.68	1.52	2.92	3.0	-4.65	-1.01	
	Hoist Eyes	7.0	175.0	41.94	49.44	40.22	-281.54	-13.56	-49.44	-18.08	1.28	16.97	2.15	19.12	3.73	18.75	4.16	.75	3.97	1.66	36
	Ballast Box	29.26	21.45	25.12	0.0	57.04	1668.99	140.0	0.0	-1.26	135.11	0.0	.004	.004	135.11	136.11	.004	123.58	123.68	74	-6.72
	Flight Weights	154.0	262.0	20.0	50.0 L R	62.5	9572.64	-100.54	-50.0	4.85	70.2	17.36	.13	17.49	70.33	87.56	83.65	336.26	418.77	-3.04	-14.59
Hoist Bar	453.0	152.1	—	0.0	9.75	-4416.75	9.35	0.0	48.55	0.61	—	—	—	0.61	0.61	—	—	6.44	-3.12	-32.94	

Table II

INERTIAL SUBTRACTIONS FROM X-24A PRIOR TO FIRST FLIGHT

I_x (slug-ft ²)	I_y (slug-ft ²)	I_{xz} (slug-ft ²)	I_z (slug-ft ²)
3.09	17.78	-3.42	20.27
1.88	36.75	-7.16	37.76
0.29	27.49	-5.16	25.57
1.50	36.12	-6.93	35.25
1.66	5.97	-1.10	5.84
3.79	38.00	-7.26	36.60
1.70	16.16	-2.82	16.94
1.85	36.80	-6.71	38.06
1.56	16.03	-3.08	15.22
1.64	16.14	-3.26	16.61
1.28	31.61	-6.37	30.67
0.57	32.03	-6.35	31.17
2.50	65.09	2.07	66.16
1.52	45.44	2.23	45.64
4.16	90.62	4.94	92.59
0.80	2.98	-1.01	3.00
	0.75	0.36	3.94
29.85	123.68	-0.67	123.68
	73.96	-32.94	73.17
		-1.94	6.44
	713.40	-0.77	155.30
		-7.60	32.98
		-94.35	1.97
			18.48
			12.96
			946.27

Table III
WEIGHT REMOVED PRIOR TO FIRST FLIGHT

Item	Weight (lb)	W.L.	Vertical Displacement (in.)	Vertical Moment (slug-ft ²)
Ballast weight	14.37	9.50	80.837	1162.03
	28.50	9.00	81.337	2318.11
	22.16	9.00	81.337	1802.10
	29.00	9.50	80.837	2344.27
	4.50	9.50	80.837	363.77
	28.37	9.00	81.337	2307.94
	14.37	11.50	78.837	1133.61
	28.50	10.00	80.337	2289.61
	14.37	10.25	80.087	1151.25
	14.50	9.50	80.837	1172.14
	29.00	9.75	80.587	2337.02
	28.50	9.75	80.587	2296.73
	25.00	28.87	61.467	1536.67
	25.00	28.87	61.467	1536.67
	50.00	29.37	60.967	3048.35
Dummy pilot	217.00	34.00	56.337	12225.13
Hoist eyes	7.00	40.50	49.837	348.86
Hoist eyes	7.00	41.94	49.837	338.78
Ballast box	29.26	25.12	65.217	1908.25
Hoist Bar	453.00		17.187	7785.71
Balance weights (gear up)	58.40		49.350	2882.04
Balance Weights (gear down)	37.20		49.350	1835.82
Total (gear up)	1127.82			52289.04
Total (gear down)	1106.60			51242.82

Table IV
WEIGHT ADDED PRIOR TO FIRST FLIGHT

Item	Weight	W.L.	Vert Displ	Vertical Moment
Flight Wts	154.0	20.0	70.337	10831.898

Table V
CHANGES BETWEEN FLIGHT 1 AND SECOND
VERTICAL CG TEST (FLIGHT X-2)

Item	Weight	W.L.	Vert Displ	Vertical Moment
Hydraulic manifold	-1.75	8.5	81.837	-143.215
Hydraulic manifold	-4.95	10.5	79.837	-395.193
Nosewheel change	-9.31	0.0	90.337	-341.037
Nosewheel steering	-1.28	13.0	77.337	-98.991
Camera	-3.00	68.0	22.337	-67.011
Tape recorder mount	+0.37	4.0	86.337	+31.915
Gas line	+0.58	16.0	74.337	+43.115
Washout filter	+0.56	10.0	80.337	+44.115
Battery case	+7.50	3.0	87.337	+655.028
Battery case	+7.44	3.0	87.337	+649.787
Battery case	+5.88	4.0	86.887	+507.662
Battery case	+7.50	4.0	86.337	+647.528
Total X-1 to X-2	+9.54			+1034.577

Table VI
EXPENDABLES ADDED PRIOR TO FLIGHT

	Weight (lb)	\bar{X} (in.)	\bar{Y} (in.)	\bar{Z} (in.)
Hydrogen peroxide	200.0	194.0	3.8	23.5
Cabin air	19.7	91.0	0.0	34.0
Helium	11.7	209.5	7.5	37.5
Emergency helium	2.1	167.0	0.0	17.5

APPENDIX IV

DERIVATION OF EQUATION 2

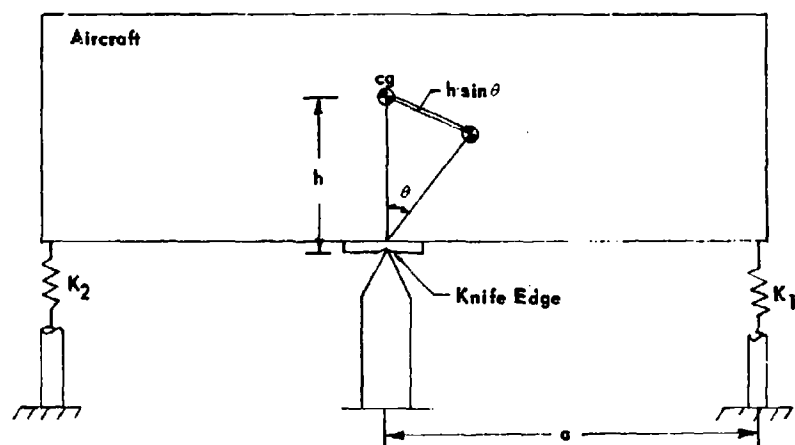


Figure 1 Schematic of Aircraft Moments About the Knife Edge

$$\Sigma T = I \ddot{\theta}$$

Taking moments about knife edge

$$\Sigma T = \overbrace{- K_T a \sin \theta}^{\text{spring depression force}} \cdot \underbrace{a}_{\text{arm}} + wh \sin \theta$$

All springs are in parallel, thus they add like resistors in series.

$$K_T = K_1 + K_2 + \dots$$

The $(wh \sin \theta)$ term is due to the inverted pendulum.

For small θ 's, $\sin \theta \doteq \theta$

$$\Sigma T = (wh - K_T a^2) \theta$$

about knife edge

$$\text{Thus } \Sigma T = I \ddot{\theta}$$

$$\theta (wh - K_T a^2) = I \ddot{\theta}$$

$$\text{Let } \theta = \theta_0 \sin \omega t$$

$$\ddot{\theta} = -\omega^2 \theta_0 \sin \omega t$$

substituting

$$\theta_0 \sin \omega t (wh - K_T a^2) = -\omega^2 \theta_0 \sin \omega t \cdot I$$

or,

$$I \omega^2 = K_T a^2 - wh$$

Thus the inertia about the knife edge is

$$I_{KE} = \frac{K_T a^2 - wh}{\omega^2}$$

To get inertia about cg, simply use transfer theorem

$$\begin{aligned} I_{cg} &= I_{\text{knife edge}} - \frac{w}{g} (h)^2 \\ &= I_{KE} - mh^2 \end{aligned}$$

APPENDIX V **X-24A MASS PROPERTIES COMPUTER** **PROGRAM LISTOUT**

593. - EFN SOURCE STATEMENT - IFN(S) -

10/16/71

```

INTEGER PART(10)
COMMON FINAL(9), ADDAT(3), PART, POXIN(72), WALIN(72), WTR(2)
COMMON POXZ(72), WALZ(72), POX(4), WAL(4), DELT, RATE1, RATE2
COMMON Z, ZAP, FA
DIMENSION COMP(4), TYTLE(8), START(8)
DATA TYTLE/2HMT, 4X C3, 4HY C6, 4HZ C3, 3HIX, 3HIYY, 3HIZZ, 3HIXZ/
103 FORMAT(1F10.2)
005 FORMAT ('(2H *) / 1X, 59(2H *) // 20X, 23HFINAL RESULTS FOR THE , 5A4 /')
006 FORMAT (' 1X, 5A6, 4F12.2, 4F12.4, 4X, 1A4')
000 FORMAT ('(2H *) / 1X, 59(2H *) // 42X,
114HSTARTING WEIGHT, C63, , AND INERTIA//')
003 FORMAT ('1X, 15HCOMPONENT TITLE, 14X, 6HWEIGHT, 6X, 6HX DIST, 5X, 6HY DIST,
15X, 44Z DIST, 4X, 34IXX, 9X, 3HIYY, 9X, 3HIZZ, 9X, 3HIXZ, 29X,
22H13, 15X, 6HINCHES, 2X, 6HINCHES, 4X, 6HINCHES, 4X, 8H1B-IN SQ, 4X,
23H13-IN SQ, 4X, 8H1B-IN SQ, 4X, 8H1B-IN SQ//')
009 FORMAT ('4A4, 1F7.1, 3F6.2, 4F7.2, 1C11')
100 FORMAT ('7F10.2')
101 FORMAT ('8F10.2')
97 FORMAT('3F10.2')
102 FORMAT('2F10.2')
READ(5,101) (WTR(I), I=1,2)
READ(5,101) (POXIN(I), I=1,8)
READ(5,100) (POXIN(I), I=9,15)
READ(5,101) (POXIN(I), I=16,71)
READ(5,101) (WALIN(I), I=1,8)
READ(5,101) (WALIN(I), I=9,15)
READ(5,101) (WALIN(I), I=16,71)
READ(5,101) (POXZ(I), I=1,8)
READ(5,100) (POXZ(I), I=9,15)
READ(5,101) (POXZ(I), I=16,71)
READ(5,101) (WALZ(I), I=1,8)
READ(5,100) (WALZ(I), I=9,15)
READ(5,101) (WALZ(I), I=16,71)
77 READ(5,97) RATE1, RATE2, ZAP
Z=0.0
READ(5,999) COMP, START, PART
IF (PART(1) .EQ. 1) GO TO 250
DO 256 K=5,9
START(K)= 144.*START(K)*32.174
256 CONTINUE
250 DO 1 I=1,8
1 FINAL(I)=START(I)
CALL TOP (COMP, PART)
WRITE (5,993)
CALL CONV(START, TYTLE)
LINE=24
IF (PART(9) .EQ. 1) GO TO 911
913 DO 5 I=1,9
5 FINAL(I)=START(I)
911 READ(5,999) COMP, ADDAT, PART
IF (ADDAT(1) .GT. 100000) GO TO 77
IF (PART(4) .EQ. 7) GO TO 67
IF (PART(6) .EQ. 8) GO TO 68
IF (PART(9) .EQ. 3) CALL LOXPR
IF (PART(9) .EQ. 4) CALL WALPR

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10/16/71

IF (PART(8) -EQ- 7) CALL LOXJT	154
IF (PART(8) -EQ- 8) CALL WALJT	157
72 IF (PART(8) -EQ- 9) CALL PLUS	161
IF (PART(8) -EQ- 1) CALL MINUS	164
IF (PART(8) -EQ- 2) CALL PERCX	167
IF (PART(8) -EQ- 5) CALL LOX	170
IF (PART(8) -EQ- 5) CALL FAL	173
IF (PART(8) -EQ- 9) GO TO 400	
GO TO 75	
420 DELT=10.	
T1=0	
T=ADDDAT(1)	
421 IF ((ABS(T-T1)) .LE. .00001) GO TO 76	
READ(5,123)ADDDAT(2)	182
FA=ADDDAT(2)	
IF ((T-T1) .GE. DELT) GO TO 51	
DELT=(T-T1)	
51 T1=T1+DELT	
WRITE (5,122) T,DELT	187
CALL PERCX	188
CALL LOX	190
CALL FAL	192
IF (LINE +25 .GT. 50) CALL TOPA(LINE)	195
LINE = LINE +25	
WRITE (5,995) COMP	197
CALL CONV (FINAL,TITLE)	199
GO TO 401	
58 FA=ADDDAT(2)	
CALL WALDEL	202
GO TO 72	
57 FA=ADDDAT(2)	
CALL LOXDEL	205
GO TO 72	
75 IF (PART(9) -EQ- 9) GO TO 911	
IF (LINE +25 .GT. 50) CALL TOPA(LINE)	211
LINE = LINE +25	
WRITE (5,995) COMP	213
CALL CONV (FINAL,TITLE)	215
76 IF (PART(7) -EQ- 5) GO TO 913	
IF (PART(10) -EQ- 3) GO TO 911	
STOP	
END	

601 - EFN SOURCE STATEMENT - IFN(S) -

10/18/71

```

SUBROUTINE MINUS
COMMON FINAL(8),ADDAT(8),PART,POXIN(72),WALIM(72),WTR(2)
COMMON POXZ(72),WALZ(72),POX(4),WAL(4),DELT, RATE1,KATE2
COMMON Z,ZAP,FA
DIMENSION CHAGE(8)
CHAGE(1)=FINAL(1)-ADDAT(1)
DO 15 I=2,4
1A CHAGE(I)=(FINAL(I)*FINAL(1)-ADDAT(I)*ADDAT(1))/CHAGE(1)
DO 15 I=5,7
CHAGE(I)= FINAL(I)-ADDAT(I)
DO 15 J=2,4
IF (J+3 .EQ. 1) GO TO 15
CHAGE(I)= CHAGE(I)+FINAL(I)*((CHAGE(J)-FINAL(J))*2)-ADDAT(I)*
((ADDAT(J)-CHAGE(J))*2)
15 CONTINUE
CHAGE(8)=FINAL(8)-ADDAT(8)-FINAL(1)*(CHAGE(2)-FINAL(2))*(FINAL(4)-
CHAGE(4))+ADDAT(1)*(CHAGE(2)-ADDAT(2))*(ADDAT(4)-CHAGE(4))
DO 5 I=1,8
5 FINAL(I)=CHAGE(I)
RETURN
END

```

602 - EFN SOURCE STATEMENT - IFN(S) -

10/18/71

```

SUBROUTINE PLUS
COMMON FINAL(8),ADDAT(8),PART,POXIN(72),WALIM(72),WTR(2)
COMMON POXZ(72),WALZ(72),POX(4),WAL(4),DELT, RATE1,KATE2
COMMON Z,ZAP,FA
DIMENSION CHAGE(8)
CHAGE(1)=FINAL(1)+ADDAT(1)
DO 4 I=2,4
4 CHAGE(I)=(FINAL(I)*FINAL(1)+ADDAT(I)*ADDAT(1))/CHAGE(1)
DO 5 I=5,7
CHAGE(I)= FINAL(I)+ADDAT(I)
DO 5 J=2,4
IF (J+3 .EQ. 1) GO TO 5
CHAGE(I)=CHAGE(I)+FINAL(I)*((CHAGE(J)-FINAL(J))*2)+ADDAT(I)*
((ADDAT(J)-CHAGE(J))*2)
5 CONTINUE
CHAGE(8)=FINAL(8)-ADDAT(8)-FINAL(1)*(CHAGE(2)-FINAL(2))*(FINAL(4)-
CHAGE(4))+ADDAT(1)*(CHAGE(2)-ADDAT(2))*(ADDAT(4)-CHAGE(4))
DO 5 I=1,8
5 FINAL(I)=CHAGE(I)
RETURN
END

```

10/18/71

```

SUBROUTINE CONV(A,E)
  DIMENSION E(8),A(8),B(8),C(8),D(8)
999 FORMAT (1X,A4,4X,14=,1X,F 9.2,3X,5HPOUNDS,9X,F 9.4,3X,5HSLUGS,10X,
1= 9.4,3X,94KILLOGRAMS//,3(1X,A4,4X,1H=,1X,F 9.4,3X,6MINCHES,9X,
2= 9.4,3X,4FEET,11X,F 9.4,3X,6HMETERS//),4(1X,A4,4X,1H=,1X,F 9.2,
33X,12HLS-FEET SQ,5X,F 9.2,3X,12HSLUG-FEET SQ,3X,F 9.2,3X,
4114KG-METER SQ//),60(2H *)//1X,59(2H *)//)
  B(1)= A(1)* 1.
  C(1)= A(1)/32.174
  D(1)= A(1)* .4535
  DO 1 I=2,4
  B(I)= A(I)* 1.
  C(I)= A(I)/12.
1 D(I)= A(I)* .7254
  DO 2 I=5,9
  B(I)= A(I)/166.
  C(I)= A(I)/(32.174*144.)
2 D(I)= A(I)* .0029255
  WRITE (5,999) (E(I),B(I),C(I),D(I),I=1,8)
  RETURN
END

```

25

10/18/71

```

SUBROUTINE TOP (9,L)
  DIMENSION A(4),L(12),3(4),K(6)
999 FORMAT (141,19X,344WEIGHT,C.G.,AND INERTIA DATA FOR ,4A4,3X,211,
11H/,211,'4/,211,21X ,4HPAGE,14//)
  DO 1 I=1,6
  A(I)=B(I)
  C(I)=L(I)
1 CONTINUE
  IPAGE=1
  WRITE (5,999) A,(K(I),I=1,6),IPAGE
  RETURN
  ENTRY TOPA(LINE)
  IPAGE=IPAGE+1
  WRITE (5,999) A,(K(I),I=1,6),IPAGE
  LINE=1
  RETURN
END

```

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605.

- EFV SOURCE STATEMENT - IFN(S) -

10/18/71

```

SUBROUTINE PEROX
INTEGER PART(10)
COMMON FINAL(9),ADDAT(8),PART,POXIN(72),WALIN(72),WTR(2)
COMMON POXZ(72),WALZ(72),POX(4),WAL(4),DELT, RATE1,RATE2
COMMON Z,ZAP,FA
DIMENSION COMP(4)
IF(PART(7),NE, 0) GO TO 1
RATE3=7.1
GO TO 50
1 IF(PART(7),NE, 1) GO TO 2
RATE3=0.31
GO TO 50
2 IF(PART(7),NE, 2) GO TO 3
RATE3=0.38
GO TO 50
3 IF(PART(7),NE, 3) GO TO 4
RATE3=0.45
GO TO 50
4 IF(PART(7),NE, 4) GO TO 5
RATE3=0.51
GO TO 50
5 IF(PART(7),NE, 5) GO TO 50
RATE3=0.013
50 DMASS=RATE3*DELT
ADDAT(1)=DMASS
ADDAT(2)=194.0
ADDAT(3)=-3.8
ADDAT(4)=23.5
CALL MINUS
RETURN
END

```

26

10/18/71

```

SUBROUTINE LOX
  INTEGER PART(1)
  COMMON FINAL(8), ADDAT(8), PART, POXIN(72), WALIN(72), WTR(2)
  COMMON POXZ(72), WALZ(72), POX(4), WAL(4), DELT, RATE1, RATE2
  COMMON Z, ZAP, FA
  DIMENSION COMP(4)
250 FORMAT (1F10.2)
251 FORMAT(10X,14HLOX REMAINING=,1F10.2)
  POXX=WTR(1)
  IF(PART(7) .NE. 1) GO TO 1
  RATE3=RATE1
  GO TO 50
1 IF(PART(7) .NE. 2) GO TO 2
  RATE3=2.*RATE1
  GO TO 50
2 IF(PART(7) .NE. 3) GO TO 3
  RATE3=3.*RATE1
  GO TO 50
3 IF(PART(7) .NE. 4) GO TO 50
  RATE3=4.*RATE1
50 ADDAT(1)=POX(1)
  ADDAT(2)=POX(2)
  ADDAT(3)=POX(3)
  ADDAT(4)=POX(4)
  CALL MINUS
  DMASS=RATE3*DELT
  WTR(1)=POXX-DMASS
  WRITE(5,251) WTR(1)
  CALL TABINT(WTR(1), ADDAT(2), FA, 8, 7, POXIN(1), 1)
  WRITE(5,251) ADDAT(2)
  CALL TABINT(WTR(1), ADDAT(4), FA, 8, 7, POXZ(1), 1)
  WRITE(5,251) ADDAT(4)
  ADDAT(1)=WTR(1)
  POX(1)=ADDAT(1)
  POX(2)=ADDAT(2)
  POX(3)=ADDAT(3)
  POX(4)=ADDAT(4)
  CALL PLUS
  RETURN
END

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607-

- EFN SOURCE STATEMENT - IFN(S) -

10/18/71

```

SUBROUTINE WALPR
INTEGER PART(1)
COMMON FINAL(8),ADDAT(8),PART,POXIN(72),WALIN(72),WTR(2)
COMMON POXZ(72),WALZ(72),POX(4),WAL(4),DELT, RATE1,RATE2
COMMON Z,ZAP,FA
DIMENSION COMP(4)
251 =FORMAT(1X,15H4ALC REMAINING=,1F10,2)
T=ADDAT(1)
WAL=WTR(2)
RATE3=0.72
DMASS=RATE3*T
WTR(2)=WAL-DMASS
WRITE(6,251)WTR(2)
ADDAT(1)=DMASS
ADDAT(2)=147.82
ADDAT(3)=19.5
ADDAT(4)=34.57
WAL(1)=WTR(2)
WAL(2)=ADDAT(2)
WAL(3)=ADDAT(3)
WAL(4)=ADDAT(4)
PART(8)=1
RETURN
END
```

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10/18/71

```

SUBROUTINE FAL
INTEGER PART(1)
COMMON FINAL(8),ADDAT(8),PART,POXIN(72),WALIN(72),WTR(2)
COMMON POXZ(72),WALZ(72),POX(6),WAL(6),DELT, RATE1,RATE2
COMMON Z,ZAP
DIMENSION COMP(4)
250 FORMAT (1F10.2)
251 FORMAT(1X,15HWALC REMAINING=,1F10.2)
WALL=WTR(2)
IF(PART(7).NE.1) GO TO 1
RATE3=RATE2
GO TO 99
1 IF(PART(7).NE.2) GO TO 2
RATE3=2.*RATE2
GO TO 99
2 IF(PART(7).NE.3) GO TO 3
RATE3=3.*RATE2
GO TO 99
3 RATE3=4.*RATE2
99 ADDAT(1)=WAL(1)
ADDAT(2)=WAL(2)
ADDAT(3)=WAL(3)
ADDAT(4)=WAL(4)
CALL MINUS
DMASS=RATE3*DELT
WTR(2)=WALL-DMASS
WRITE(5,251)WTR(2)
CALL TABINT(WTR(2),ADDAT(2),FA,8,7,WALIN(1),1)
WRITE(5,250)ADDAT(2)
CALL TABINT(WTR(2),ADDAT(4),FA,8,7,WALZ(1),1)
WRITE(5,250)ADDAT(4)
ADDAT(1)=WTR(2)
WAL(1)=ADDAT(1)
WAL(2)=ADDAT(2)
ADDAT(3)=19.5
WAL(3)=ADDAT(3)
WAL(6)=ADDAT(6)
CALL PLUS
RETURN
END

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599

- EFN SOURCE STATEMENT - IFN(S) -

10/18/71

```

SUBROUTINE WALJT
INTEGER PART(1)
COMMON FINAL(8),ADDAT(8),PART,POXIN(72),WALIN(72),WTR(2)
COMMON POXZ(72),WALZ(72),POX(6),WAL(6),DELT, RATE1,RATE2
COMMON Z,ZAP,FA
DIMENSION COMP(4)
250 FORMAT (1F10.2)
251 FORMAT(10X,15HWALC REMAINING=,1F10.2)
T=ADDAT(1)
WAL1=WTR(2)
FA=ADDAT(2)
ADDAT(1)=WAL(3)
ADDAT(2)=WAL(2)
ADDAT(3)=WAL(3)
ADDAT(4)=WAL(6)
CALL MINUS
RATE3=43.0
DMASS=RATE3*T
WTR(2)=WAL1-DMASS
WRITE(6,251)WTR(2)
CALL TABINT(WTR(2),ADDAT(2),FA,8,7,WALIN(1),1)
CALL TABINT(WTR(2),ADDAT(4),FA,8,7,WALZ(1),1)
WRITE(6,250)ADDAT(2)
WRITE(6,250)ADDAT(4)
ADDAT(1)=WTR(2)
WAL(1)=ADDAT(1)
WAL(2)=ADDAT(2)
ADDAT(3)=19.5
WAL(3)=ADDAT(3)
WAL(4)=ADDAT(4)
PART(9)=0
RETURN
END

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701

- EFV SOURCE STATEMENT - IFNCS -

10/18/71

```

SUBROUTINE LOKJT
  INTEGER PART(1)
  COMMON FINAL(8),ADJAT(8),PART,POXIN(72),WALIN(72),WTR(2)
  COMMON POXZ(72),WALZ(72),POX(4),WAL(4),DELT, RATE1,RATE2
  COMMON Z,ZAP,FA
  DIMENSION COMP(4)
200  FORMAT (1F10,2)
201  FORMAT (10X,16HBOX REMAINING=,1F10,2)
  F=ADJAT(1)
  POXY=WTR(1)
  FA=ADJAT(2)
  ADJAT(1)=POX(1)
  ADJAT(2)=POX(2)
  ADJAT(3)=POX(3)
  ADJAT(4)=POX(4)
  CALL MINUS
  RATE2=73.0
  DMASS=RATE3*T
  WTR(1)=POXY-DMASS
  WRITE(5,251) WTR(1)
  CALL TABINT(WTR(1),ADJAT(2),FA,8,7,WALIN(1),1)
  WRITE(5,250)ADJAT(2)
  CALL TABINT(WTR(1),ADJAT(4),FA,8,7,WALZ(1),1)
  WRITE(5,252)ADJAT(4)
  ADJAT(1)=WTR(1)
  POX(1)=ADJAT(1)
  POX(2)=ADJAT(2)
  ADJAT(3)=1.0
  POX(3)=ADJAT(3)
  POX(4)=ADJAT(4)
  DELT(8)=0
  RETURN
END

```

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701

- EFN SOURCE STATEMENT - IFN(S) -

10/18/71

```

SUBROUTINE LOXPP
  INTEGER PART(1)
  COMMON FINAL(R),ADDAT(8),PART,POXIN(72),WALIN(72),WTR(2)
  COMMON POXZ(72),WALZ(72),POX(4),WAL(4),DELT, RATE1,RATE2
  COMMON Z,ZAP,FA
  DIMENSION COMP(5)
251  FORMAT(10X,'AHLOX REMAINING=,2F10.2)
      POXX=WTR(1)
      T=ADDAT(1)
      DMASS=0.0
      SUB=0.0
      IF(ZAP-LE-ZAP-0.0001) GO TO 801
      AVIV=T+Z
      IF(AVIV-LE-ZAP) GO TO 802
      SUB=-31.3*(ZAP-Z)
      T=T-(ZAP-Z)
      Z=ZAP
      GO TO 371
802  SUB=-31.3*T
      Z=Z+T
      T=0.0
801  DMASS=3.84*T
      DMASS=DMASS+SUB
      ADDAT(1)=DMASS
      WTR(1)=POXX-DMASS
      WRITE(5,251) WTR(1)
      ADDAT(2)=148.55
      ADDAT(3)=-19.5
      ADDAT(4)=36.50
      POX(1)=WTR(1)
      POX(2)=ADDAT(2)
      POX(3)=ADDAT(3)
      POX(4)=ADDAT(4)
      PART(R)=1
      RETURN
END

```

11

704-

- EFV SOURCE STATEMENT - IFN(5) -

10/18/71

```

SUBROUTINE LOXDEL
INTEGER PART(10)
COMMON FINAL(8),ADDAT(8),PART,POXIN(72),WALIN(72),WTR(2)
COMMON POXZ(72),WALZ(72),POX(4),WAL(4),DELT, RATE1,RATE2
COMMON Z,ZAP,FA
DIMENSION COMP(4)
250 FORMAT (1F10.2)
T=ADDAT(1)
FA=ADDAT(2)
POXX=WTR(1)
ADDAT(1)=POX(1)
ADDAT(2)=POX(2)
ADDAT(3)=POX(3)
ADDAT(4)=POX(4)
CALL MINUS
WRITE(5,250)FA
CALL TABINT(WTR(1),ADDAT(2),FA,8,7,POXIN(1),1)
WRITE(5,250)ADDAT(2)
CALL TABINT(WTR(1),ADDAT(4),FA,8,7,POXZ(1),1)
WRITE(5,250)ADDAT(4)
ADDAT(1)=WTR(1)
POX(1)=ADDAT(1)
POX(2)=ADDAT(2)
POX(3)=ADDAT(3)
POX(4)=ADDAT(4)
PART(8)=0
RETURN
END

```

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709 - EFV SOURCE STATEMENT - IFN(S) -

10/18/71

```

SUBROUTINE WALDEL
INTEGER PART(1)
COMMON FINAL(8), ADDAT(8), PART, PCXIN(72), WALIN(72), WTR(2)
COMMON PCXZ(72), WALZ(72), PCX(4), WAL(4), DELT, RATE1, RATE2
COMMON Z, ZAP, FA
DIMENSION COMP(4)
250 FORMAT (1F10.2)
T=ADDAT(1)
FA=ADDAT(2)
WALL=WTR(2)
ADDAT(1)=WAL(1)
ADDAT(2)=WAL(2)
ADDAT(3)=WAL(3)
ADDAT(4)=WAL(4)
CAL_ MINUS
CAL_ TABINT(WTR(2), ADDAT(2), FA, 8, 7, WALIN(1), 1)
WRITE(5,250) ADDAT(2)
CAL_ TABINT(WTR(2), ADDAT(4), FA, 8, 7, WALZ(1), 1)
WRITE(5,250) ADDAT(4)
ADDAT(1)=WTR(2)
WAL(1)=ADDAT(1)
WAL(2)=ADDAT(2)
ADDAT(3)=19.5
WAL(3)=ADDAT(3)
WAL(4)=ADDAT(4)
PART(8)=0
RETURN
END

```

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APPENDIX VI
TIME HISTORIES OF FLIGHT
MASS PROPERTIES



FIGURES 1 THROUGH 18 ARE
ON THE FOLLOWING PAGES

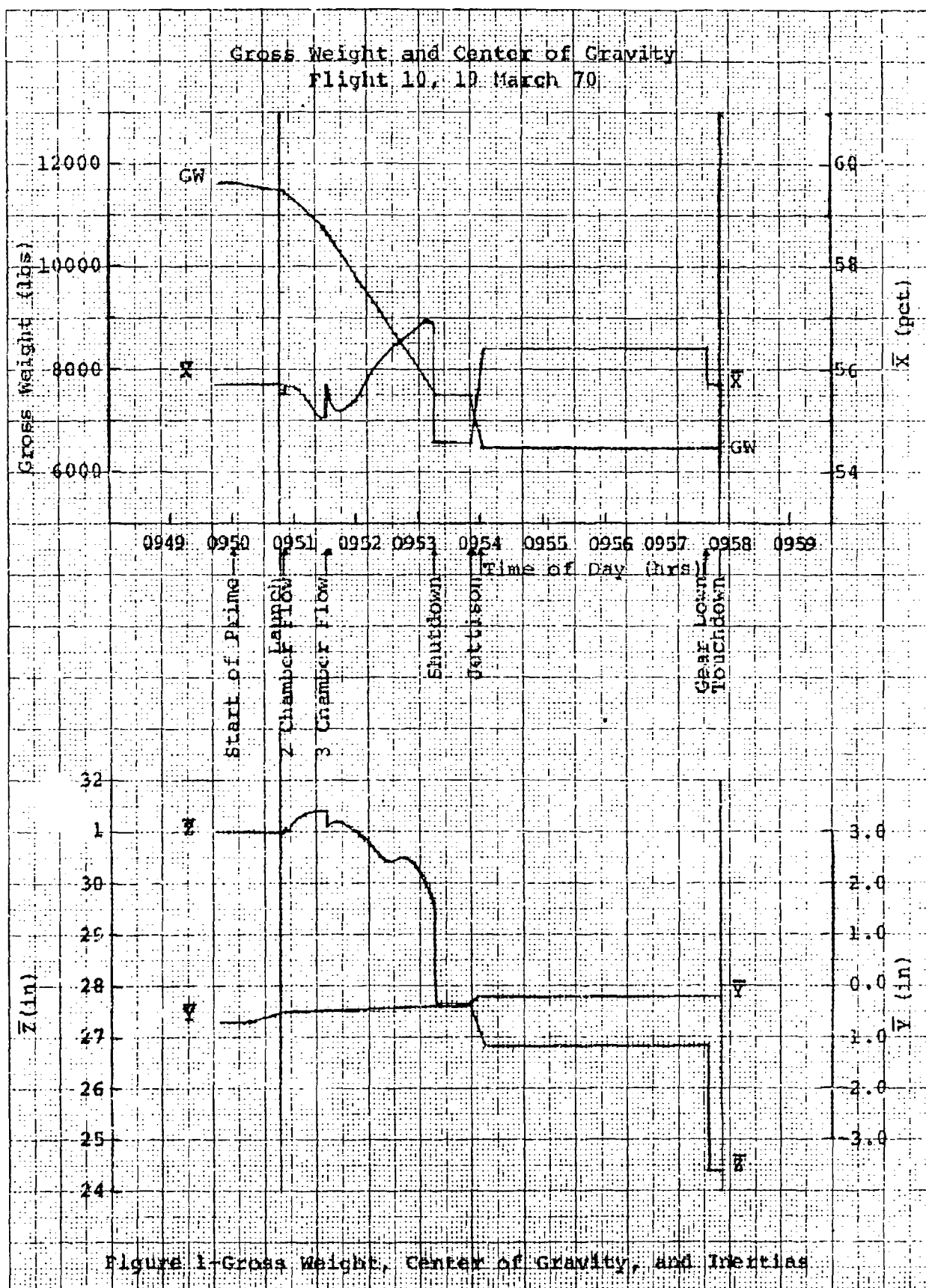
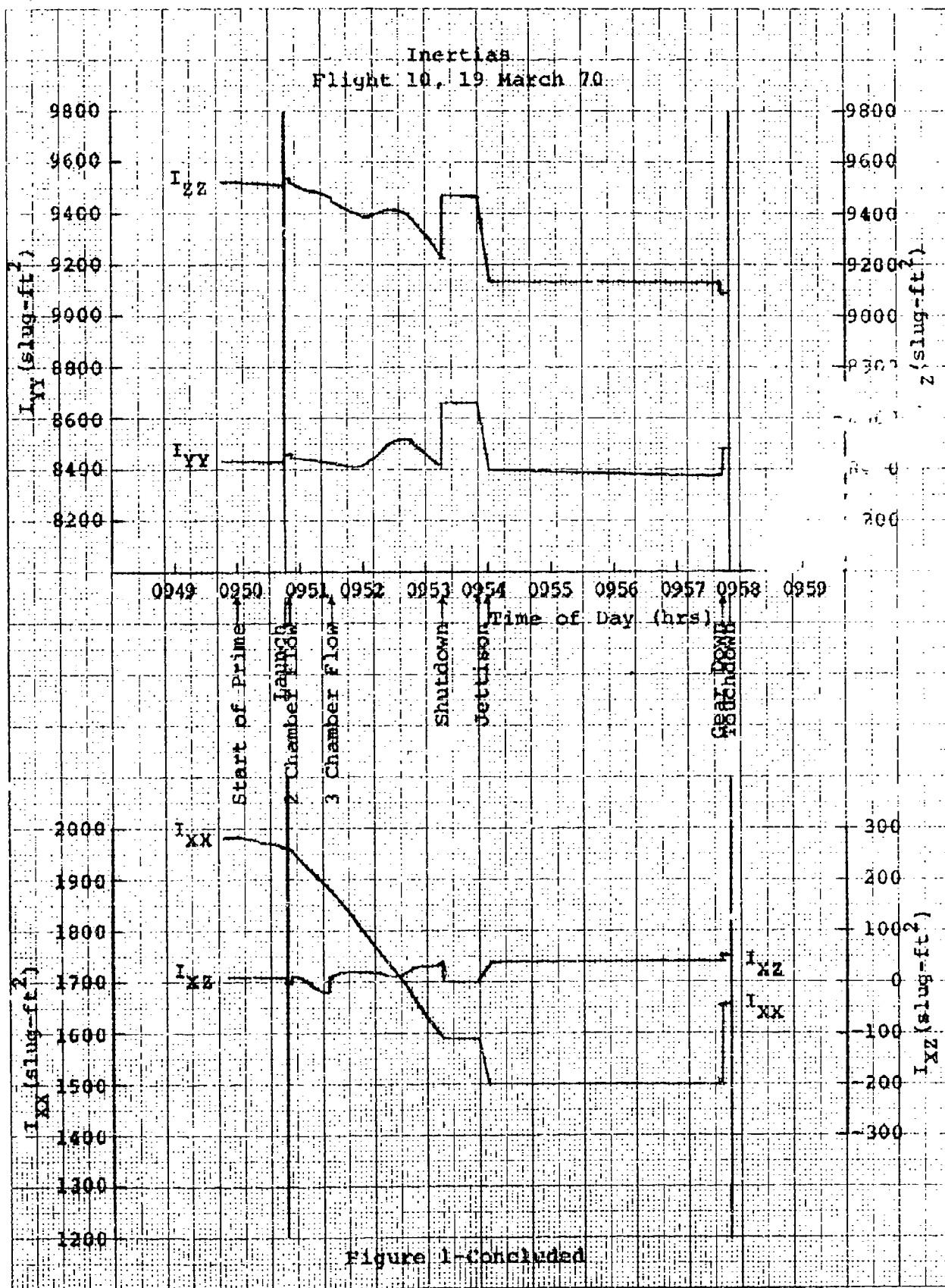


Figure 1-Gross Weight, Center of Gravity, and Inertias



Gross Weight and Center of Gravity Flight 11, 2 April 70

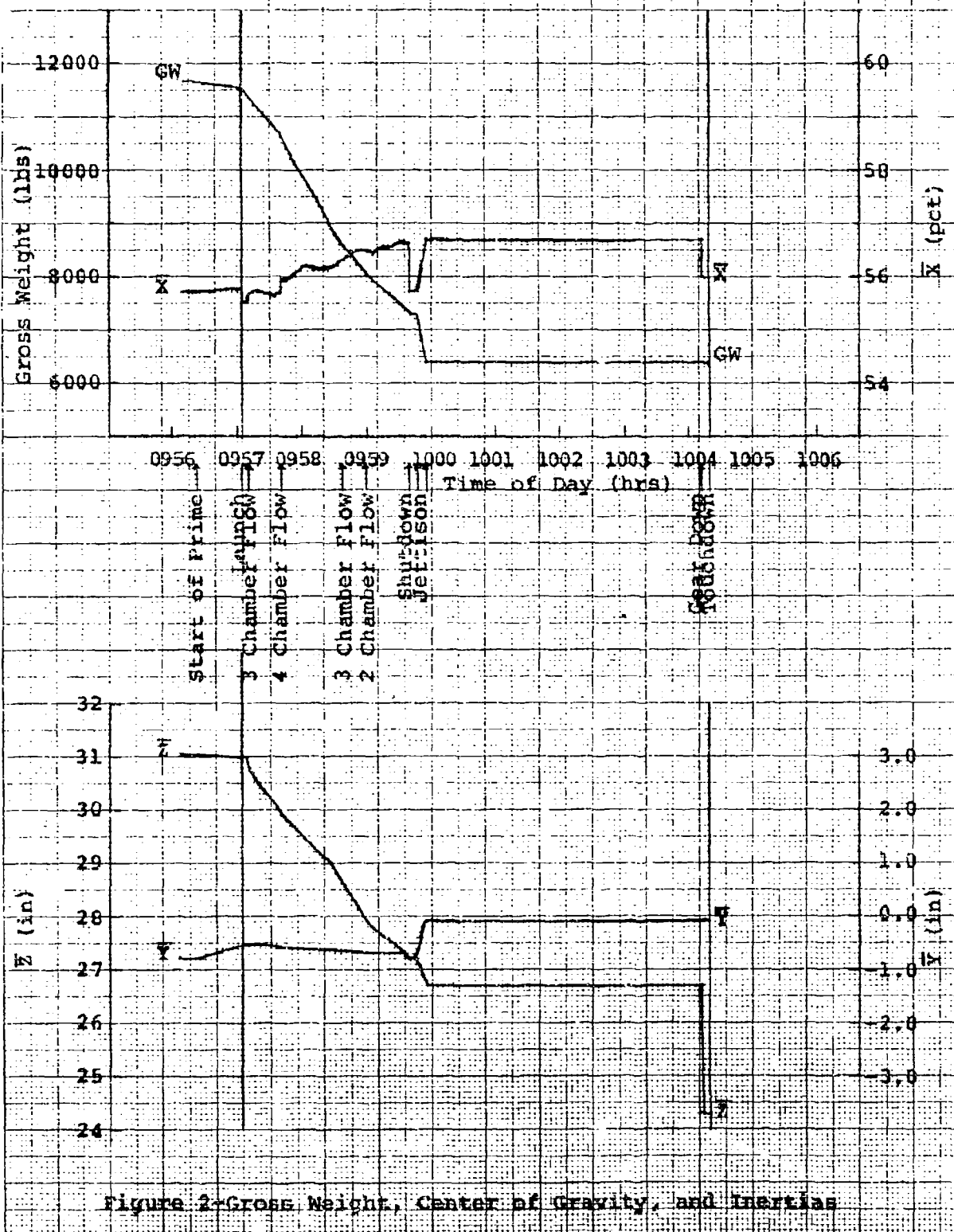
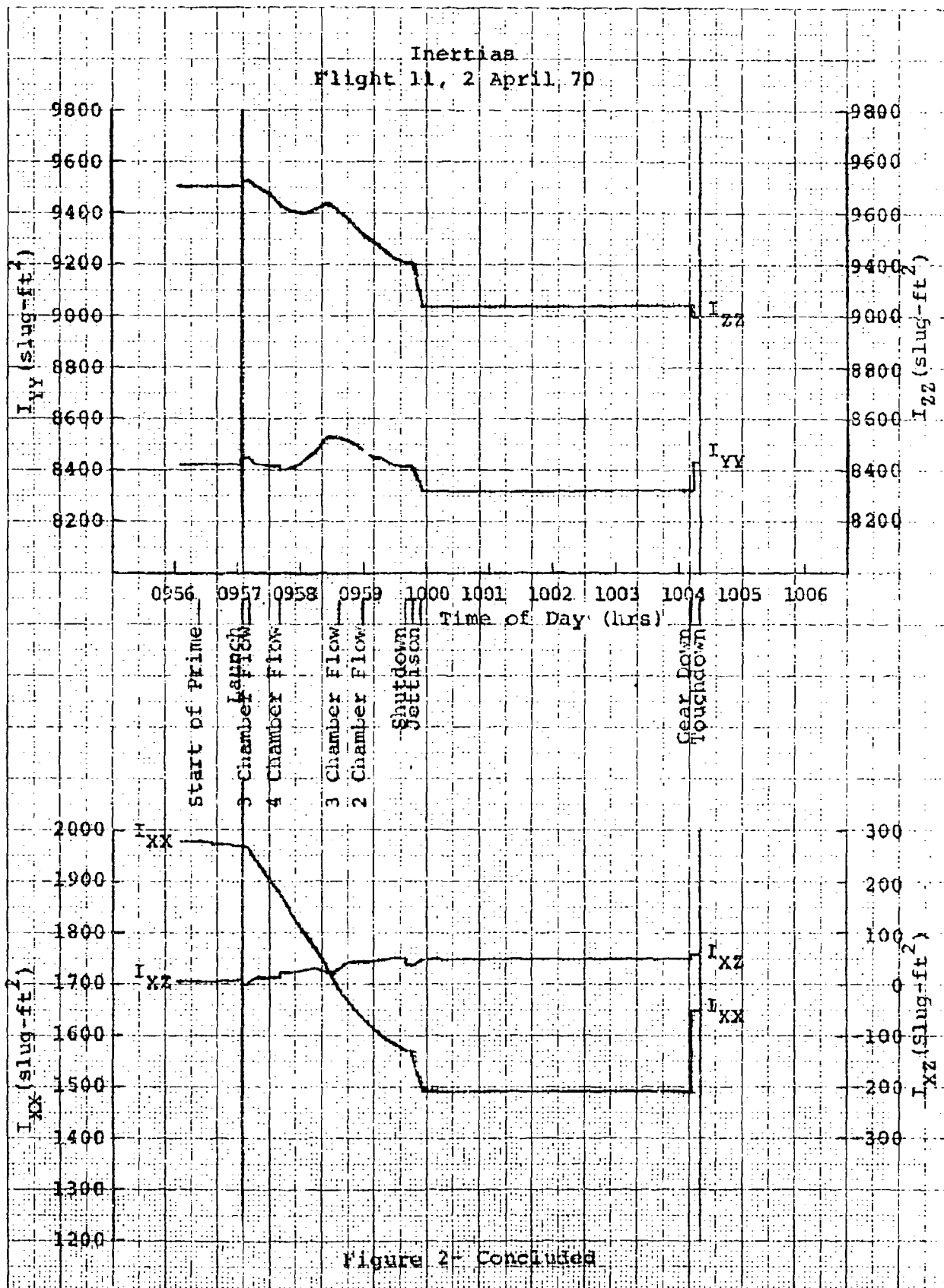


Figure 2-Gross Weight, Center of Gravity, and Inertia



Gross Weight and Center of Gravity Flight 12, 22 April 70

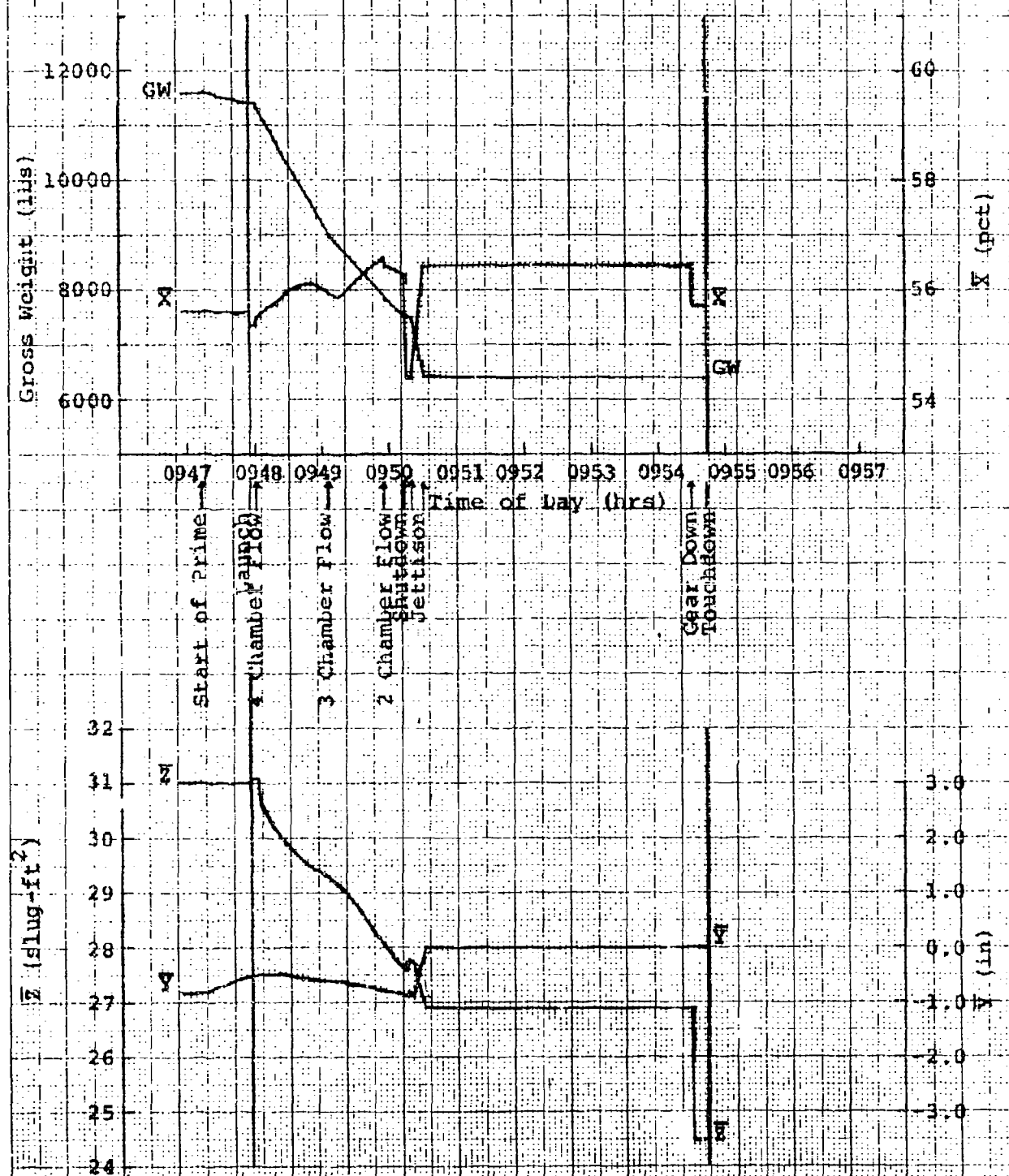
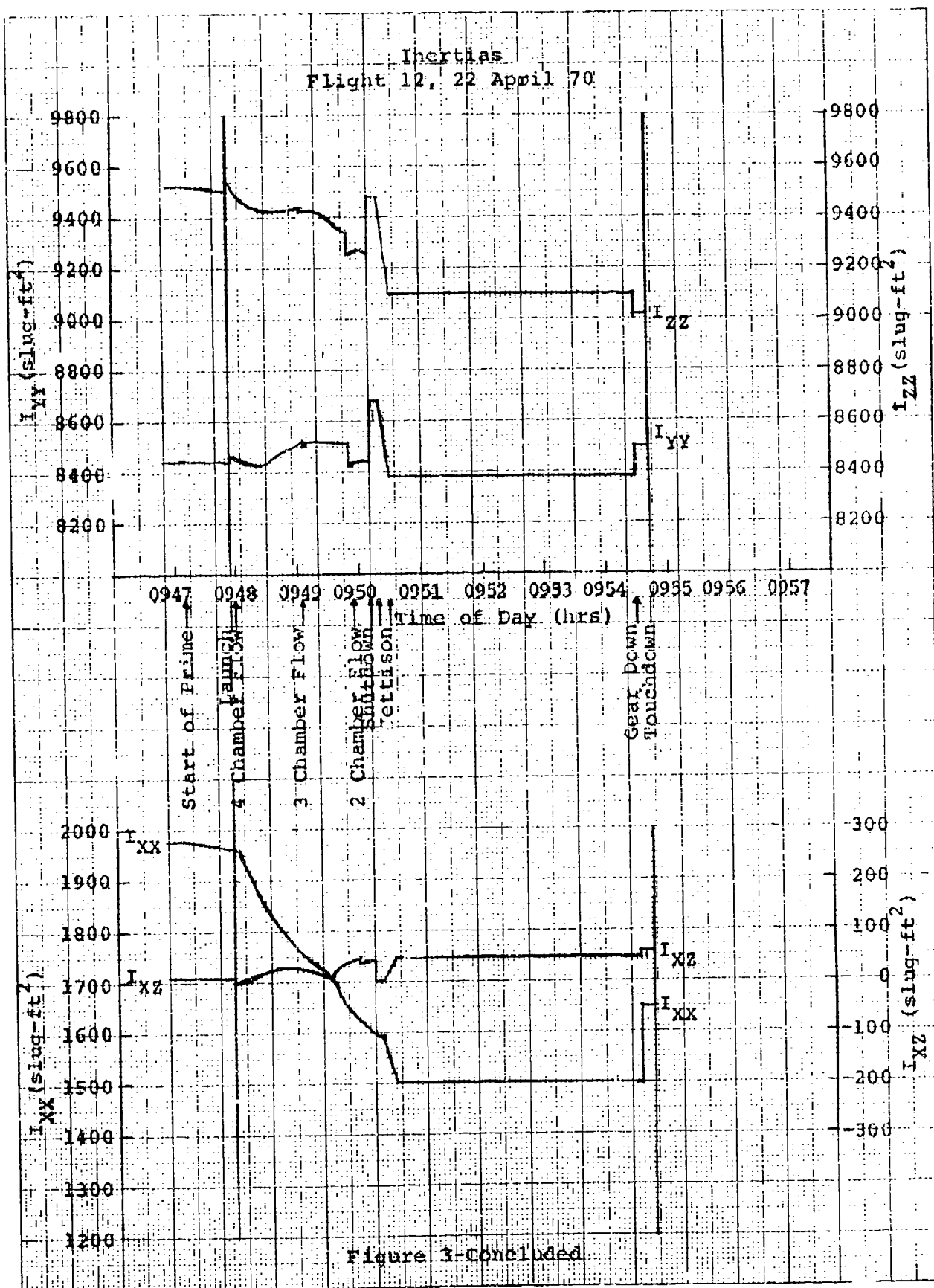


Figure 3-Gross Weight, Center of Gravity, and Inertias



Gross Weight and Center of Gravity Flight 13, 12 May 70

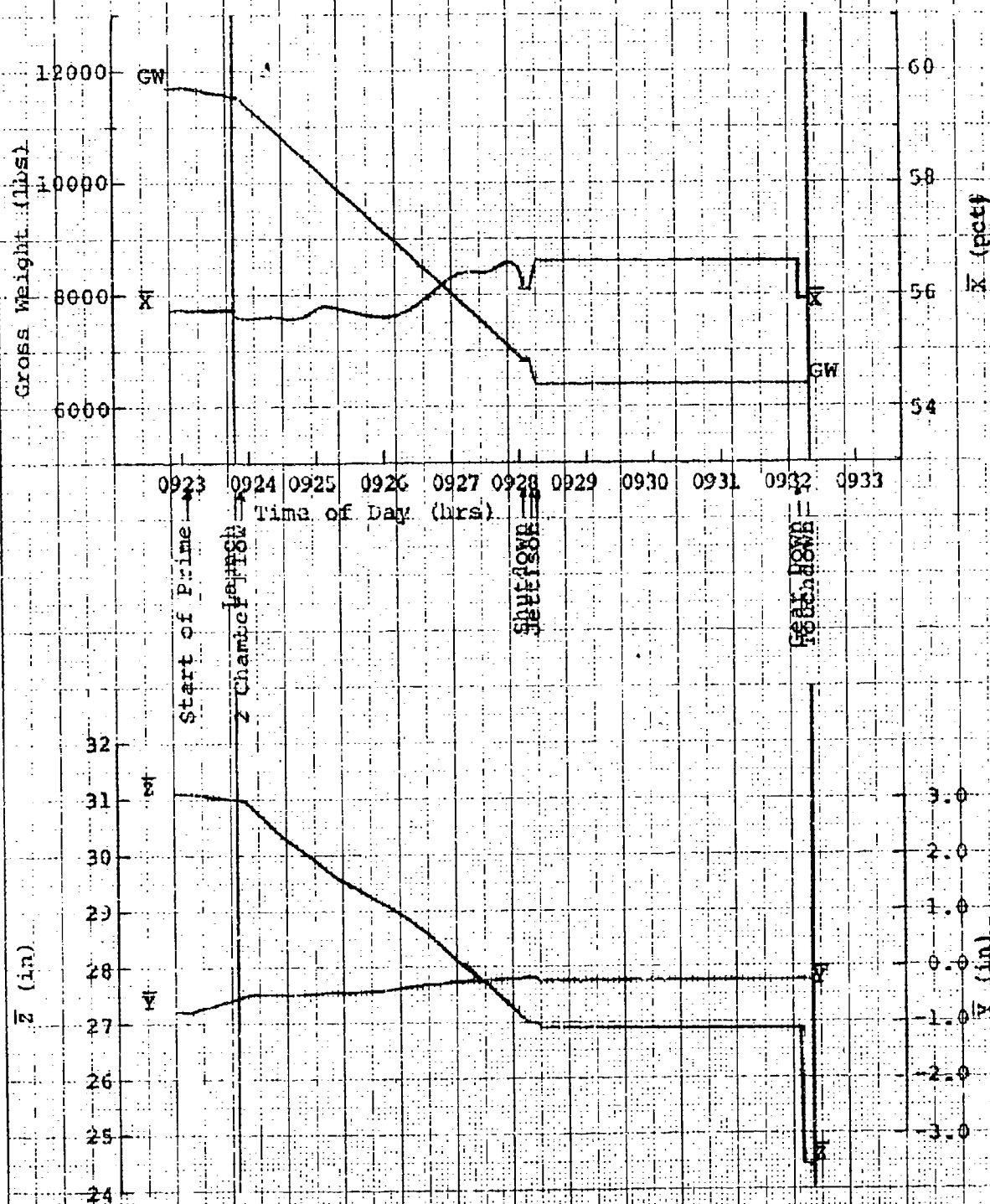
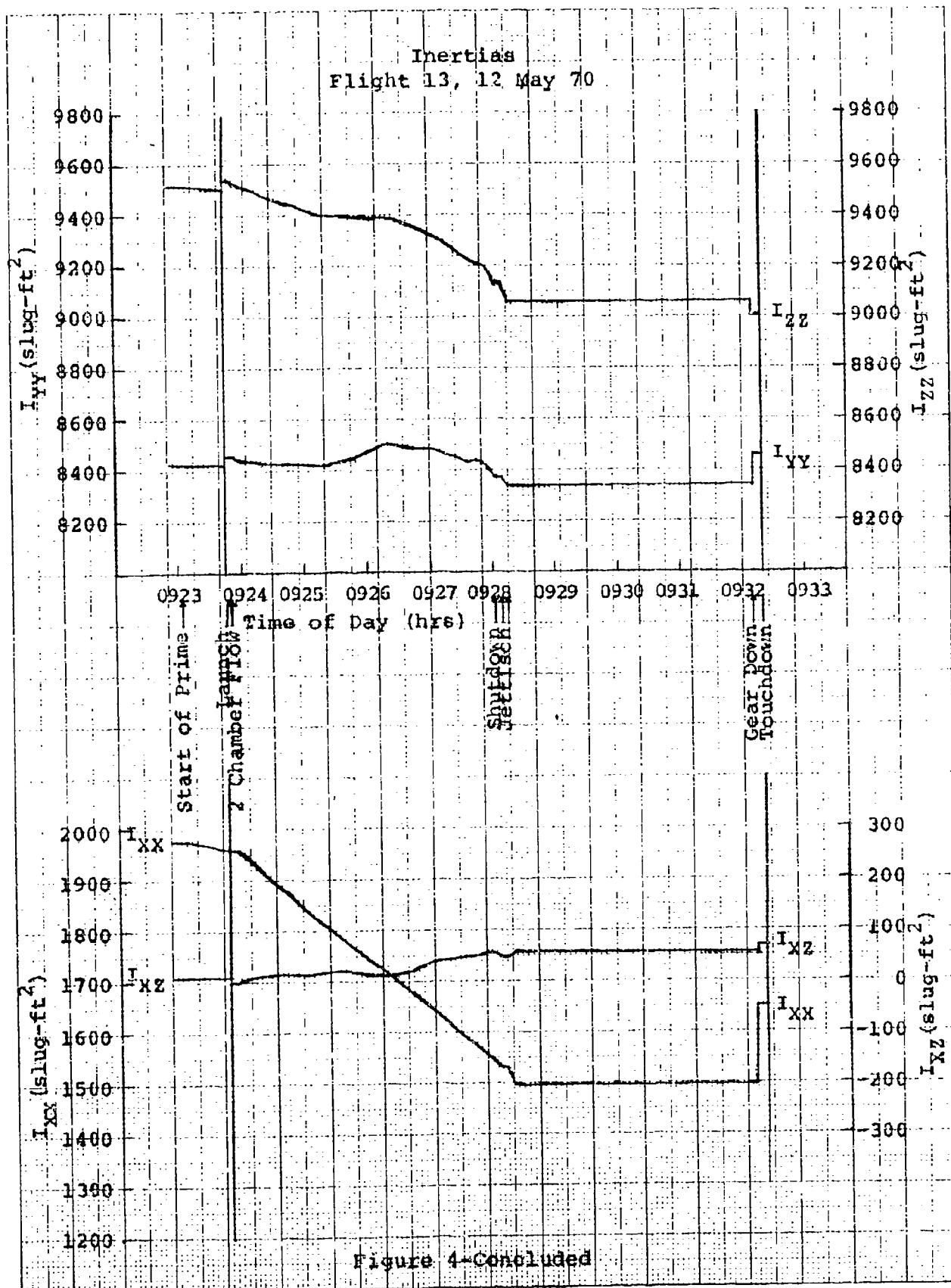


Figure 4-Gross Weight, Center of Gravity, and Inertias



Gross Weight and Center of Gravity Flight 14, 17 June 70

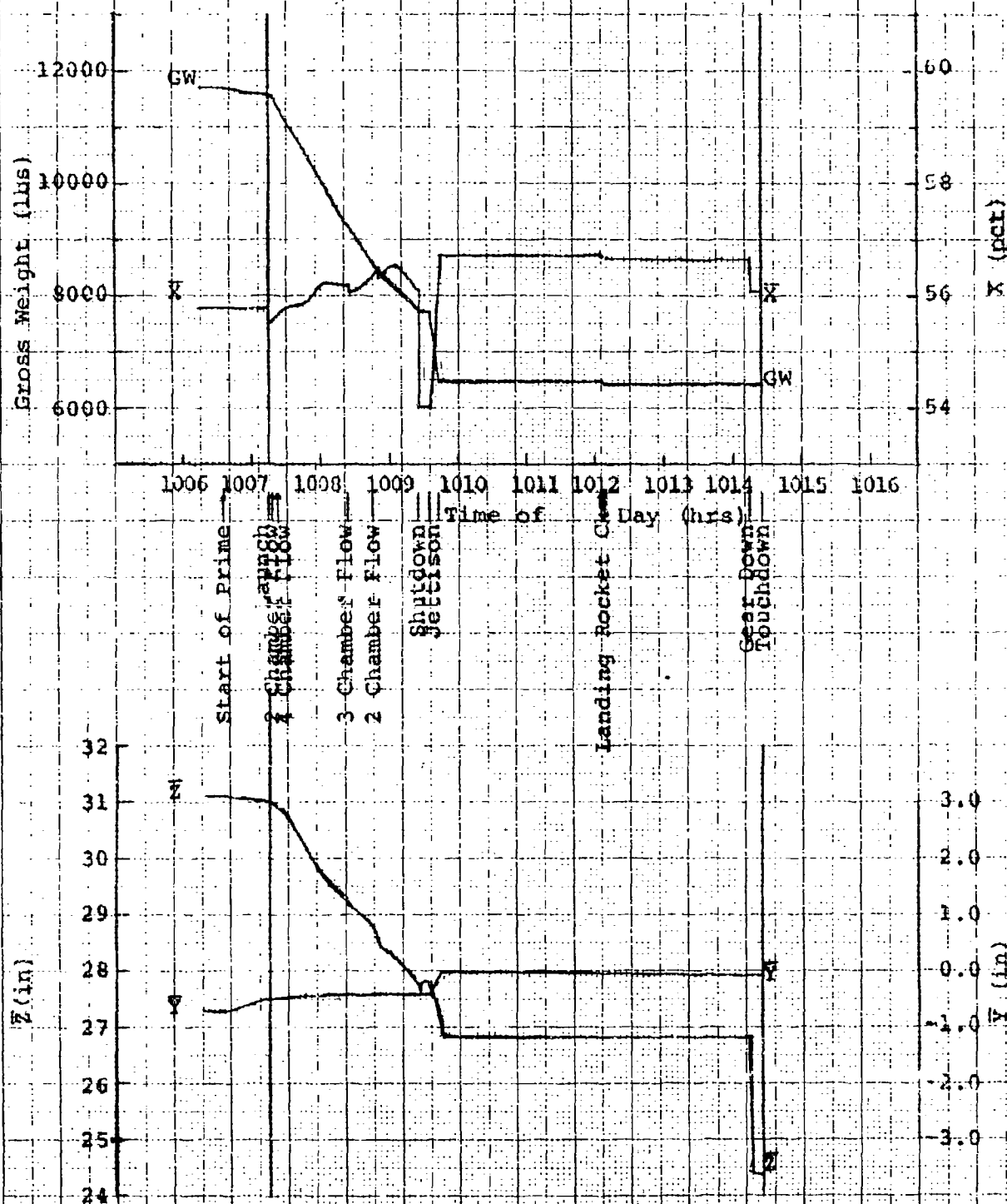
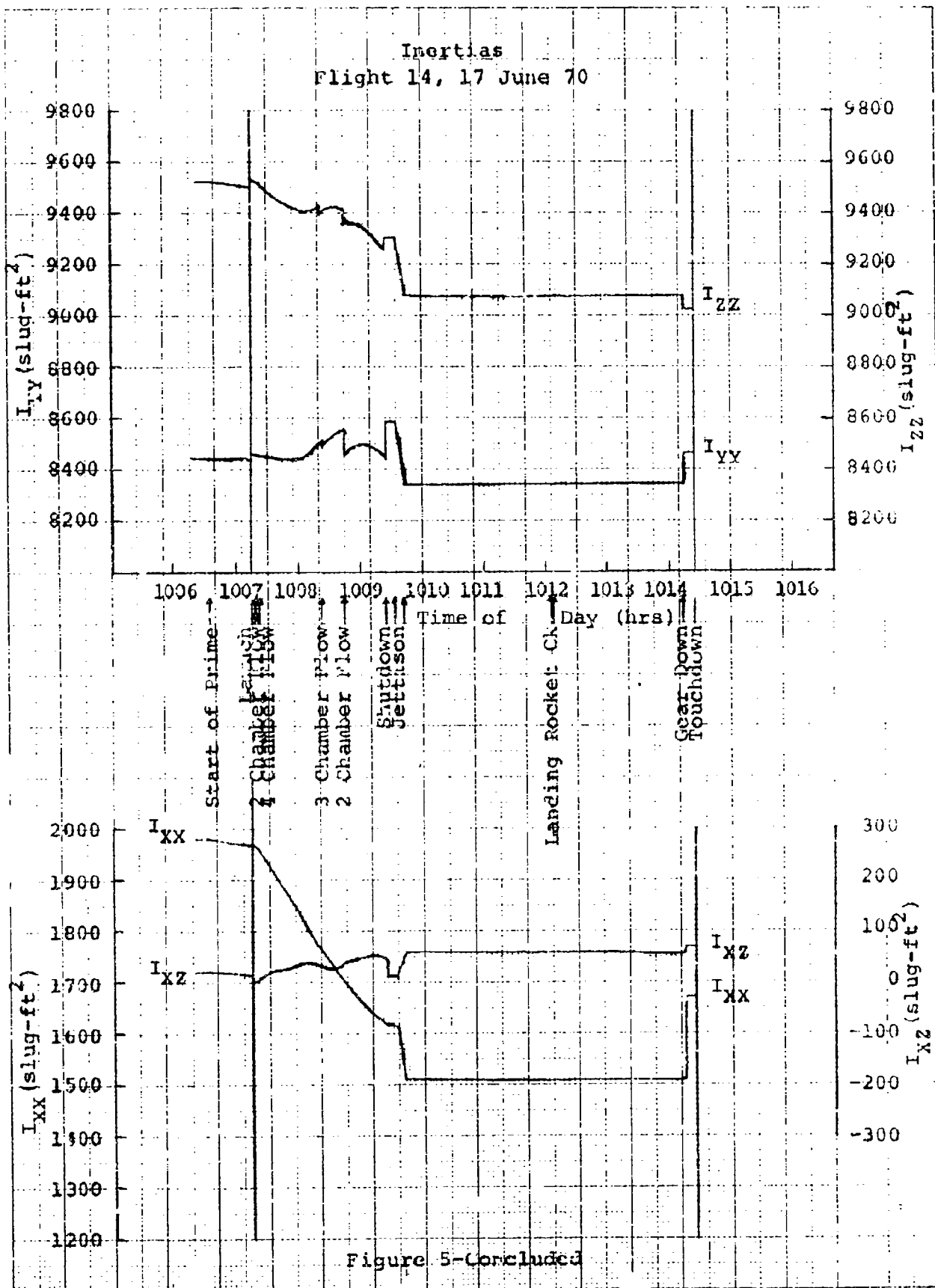


Figure 5-Gross Weight, Center of Gravity, and Inertias



Gross Weight and Center of Gravity Flight 15, 28 July 70

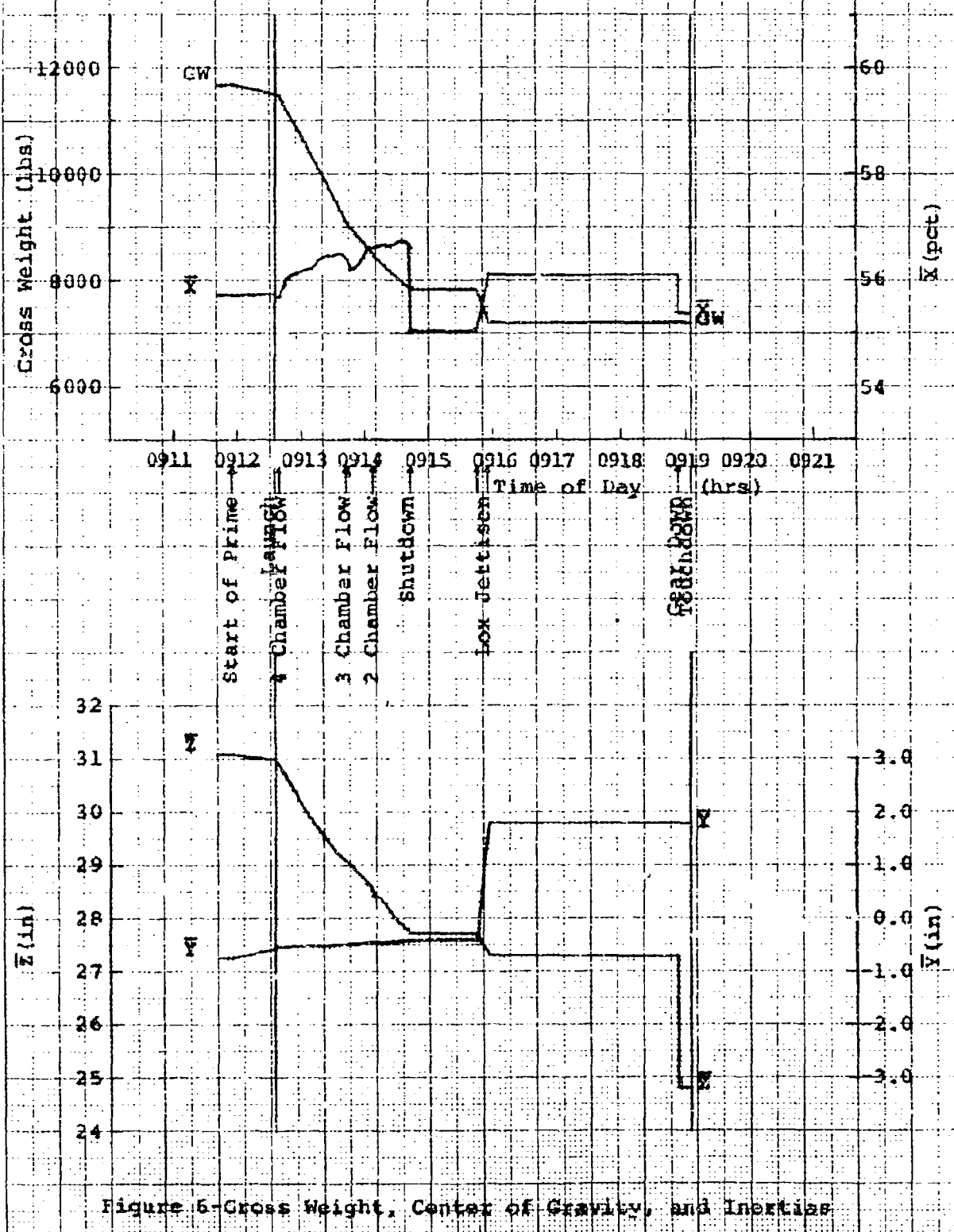
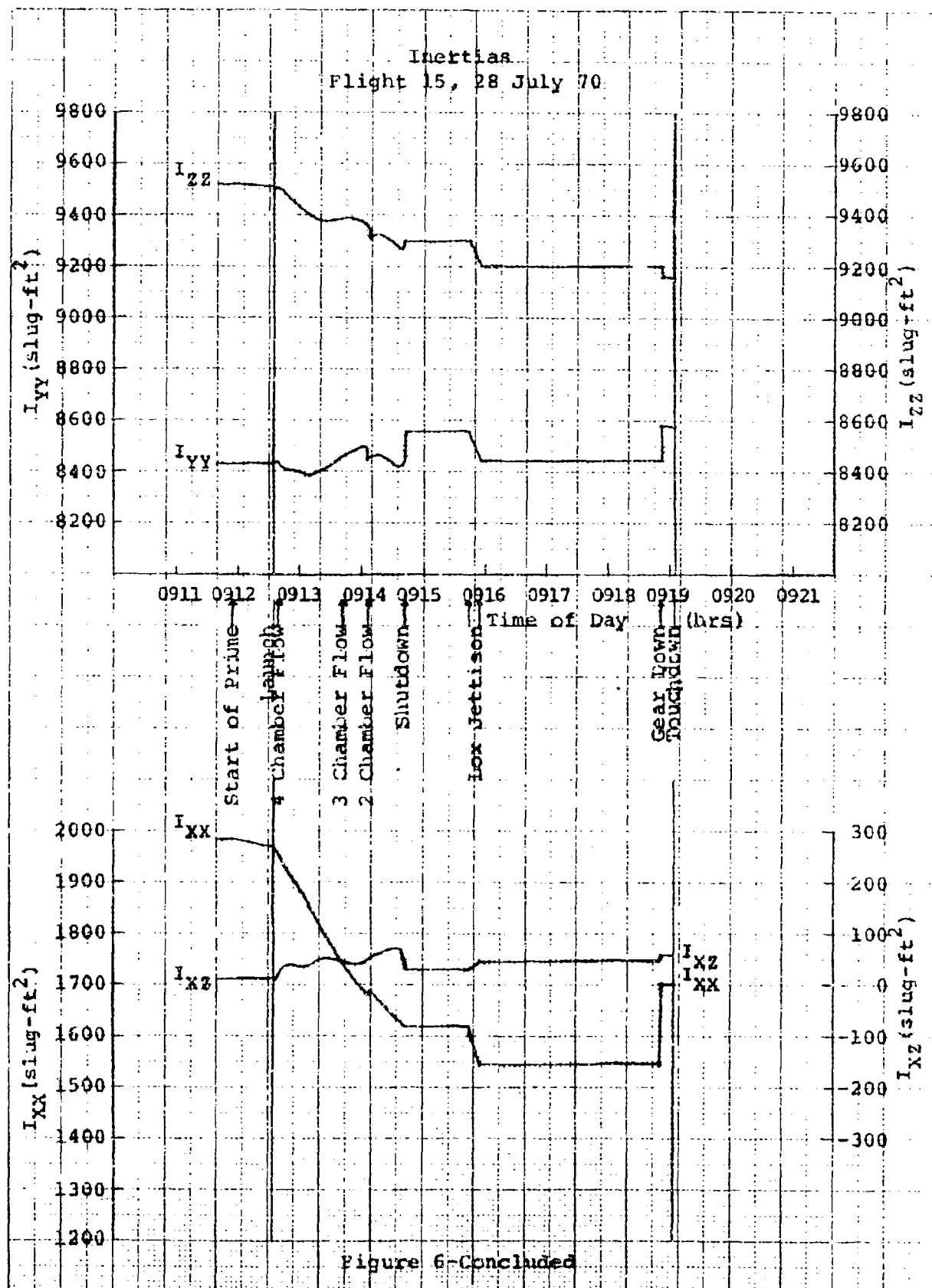
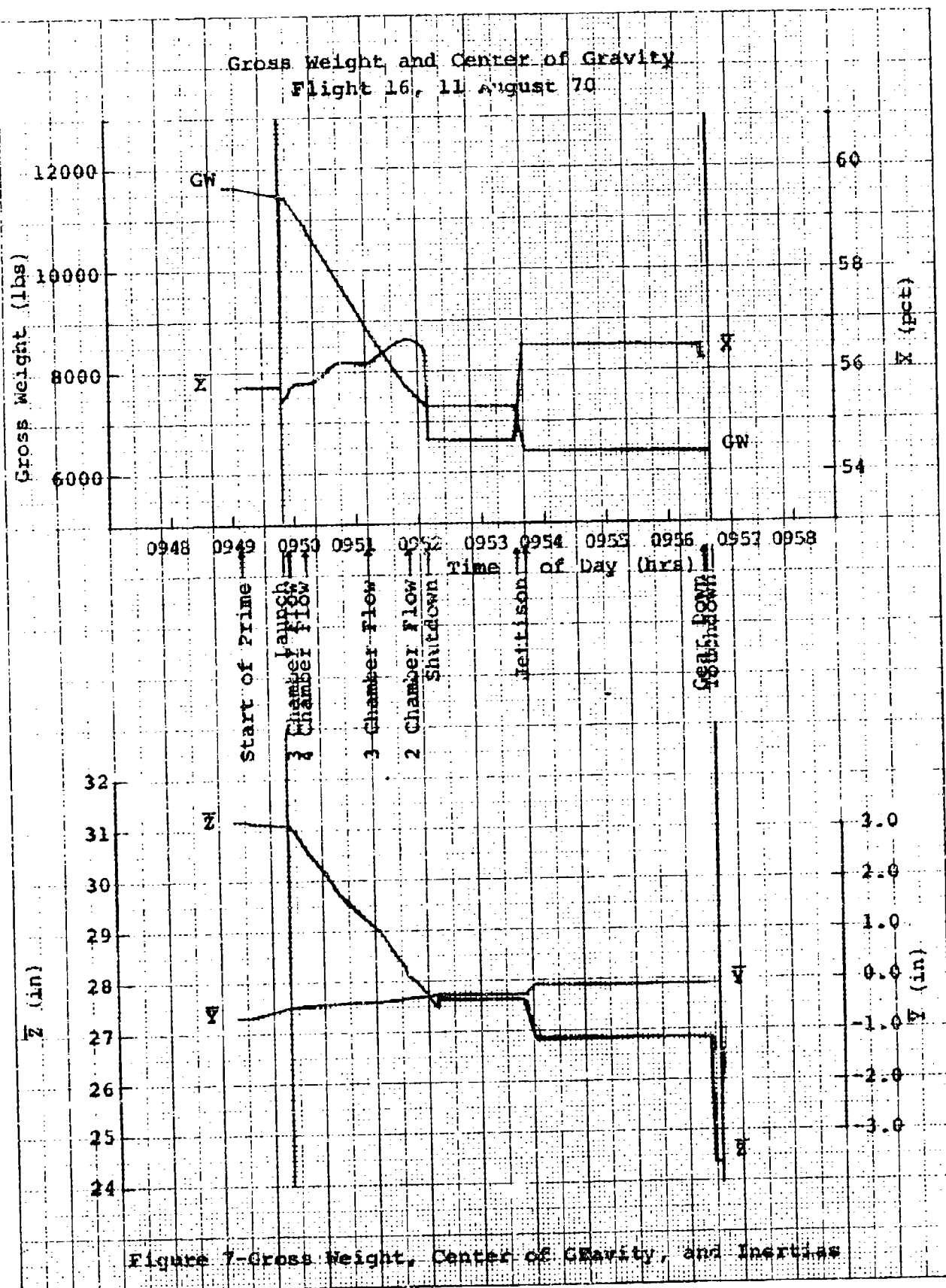
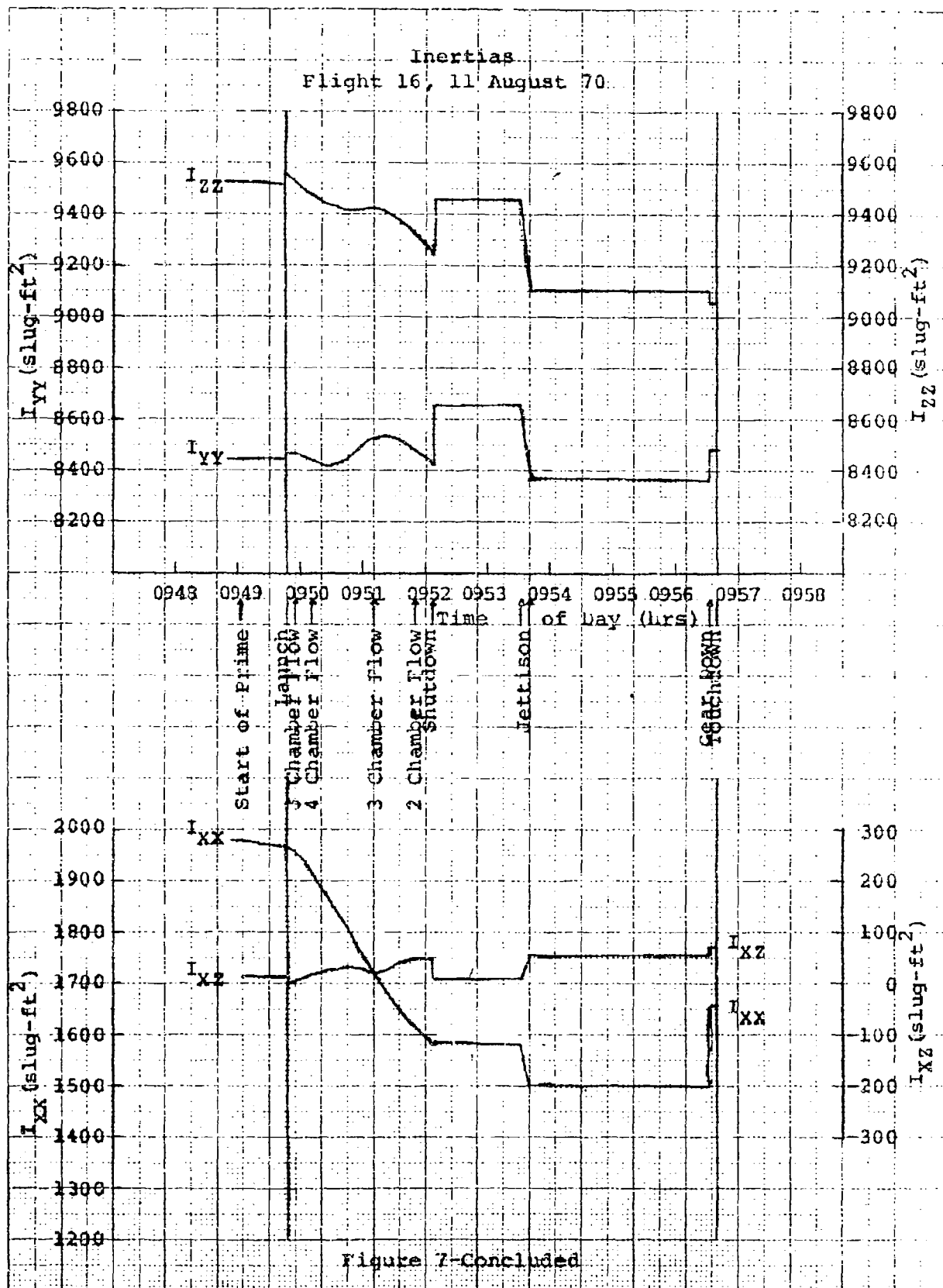


Figure 6-Gross Weight, Center of Gravity, and Inertias







Gross Weight and Center of Gravity Flight 17, 26 August 70

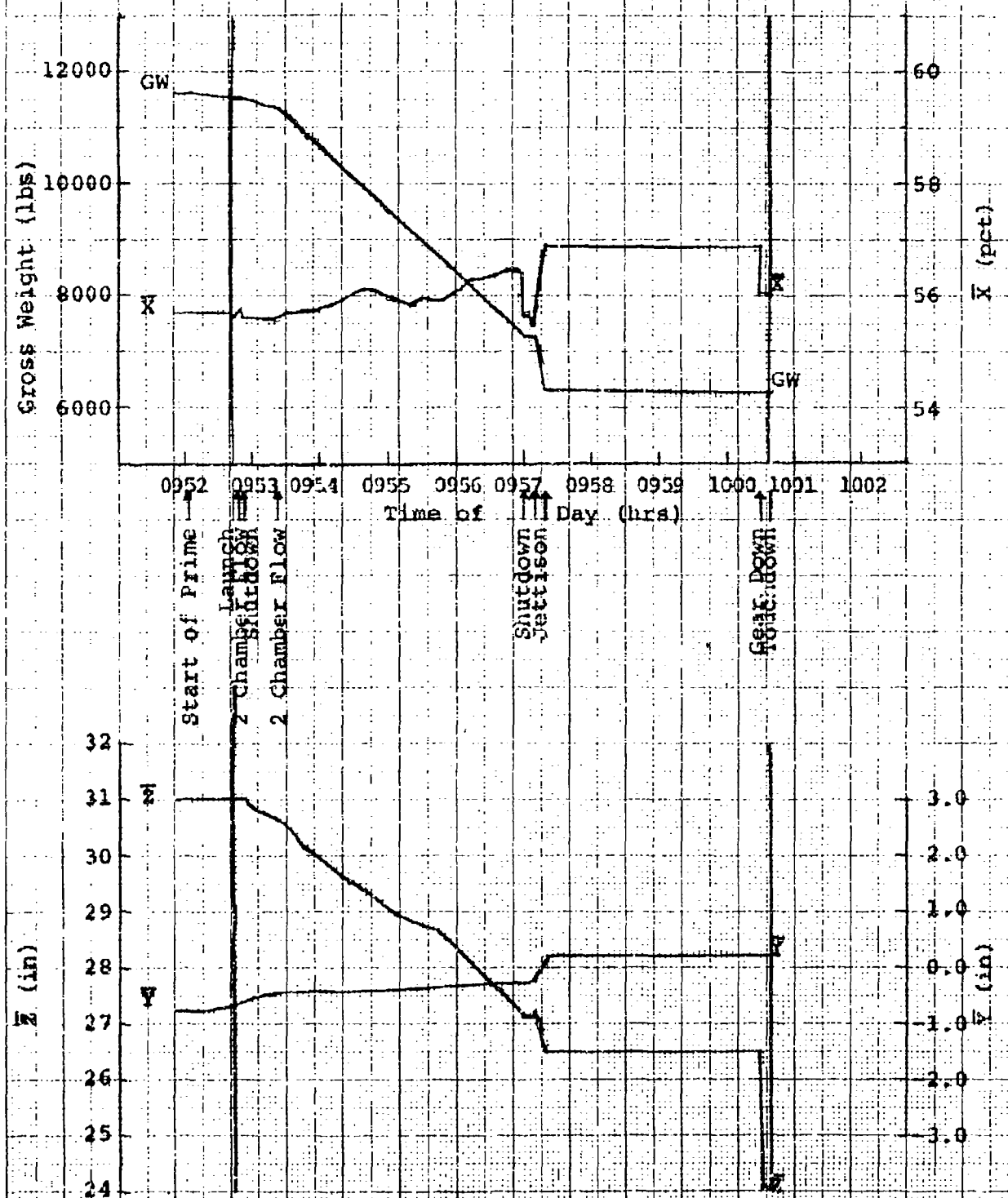


Figure 8-Gross Weight, Center of Gravity, and Inertias

Inertias Flight 17, 26 August 70

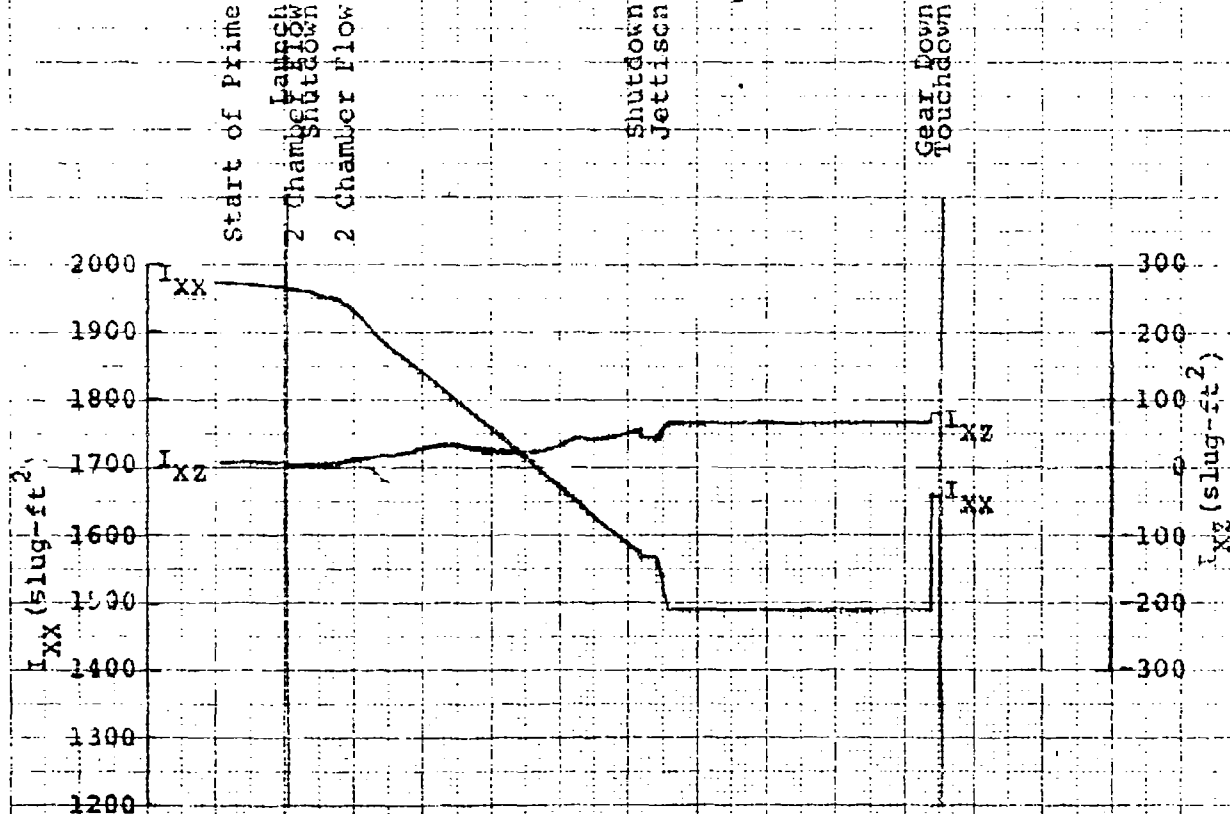
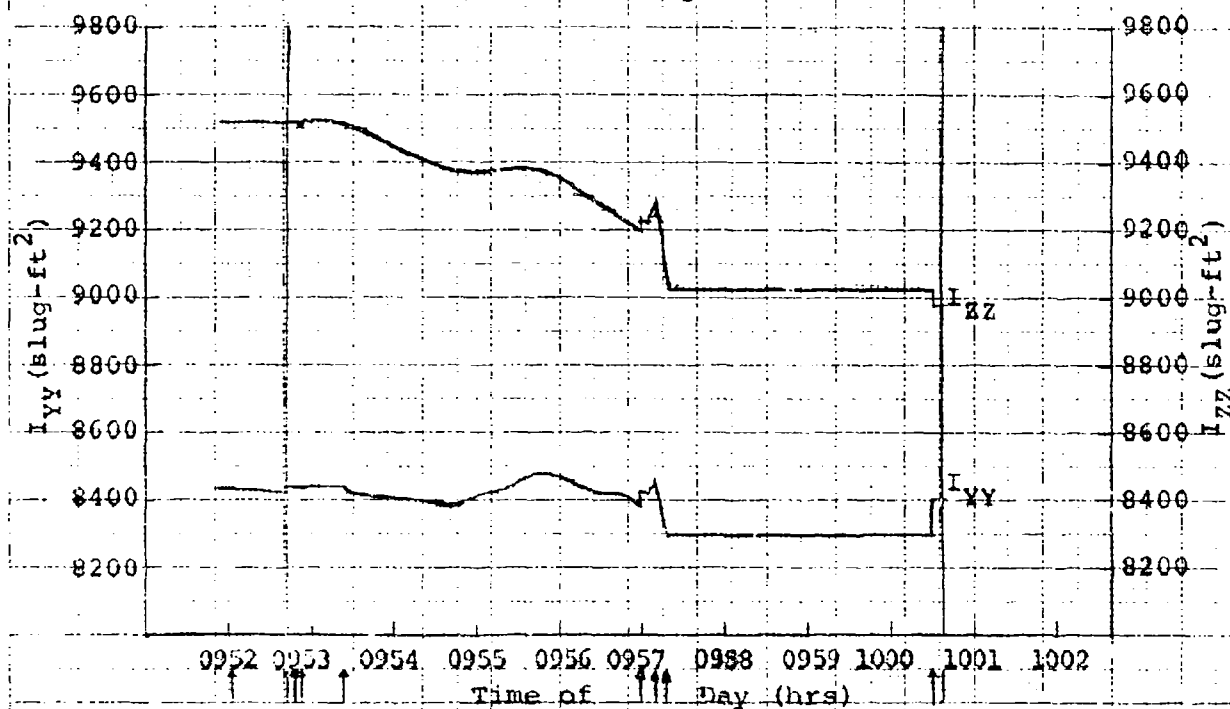


Figure B-Concluded

Gross Weight and Center of Gravity Flight 18, 14 October 70

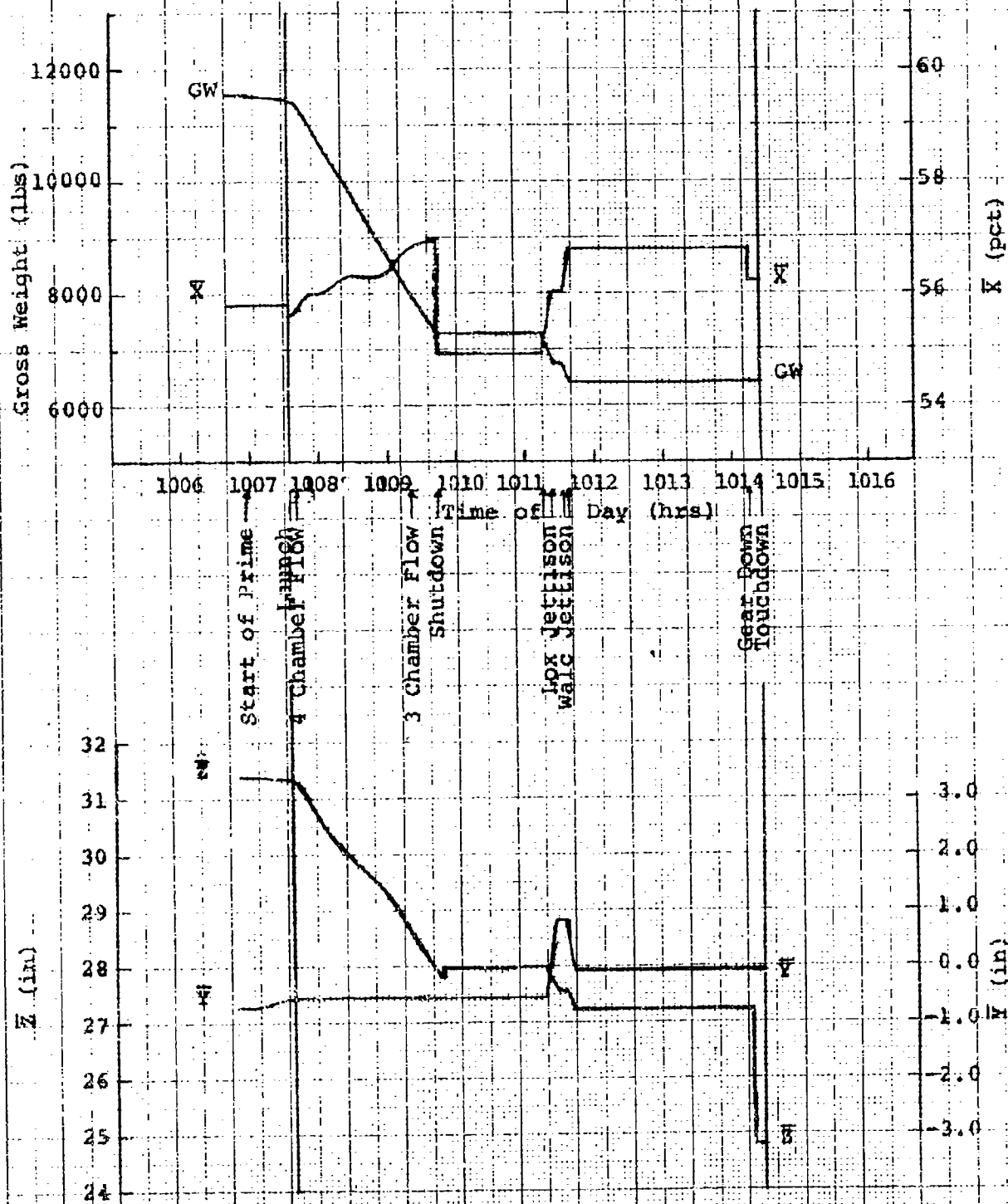
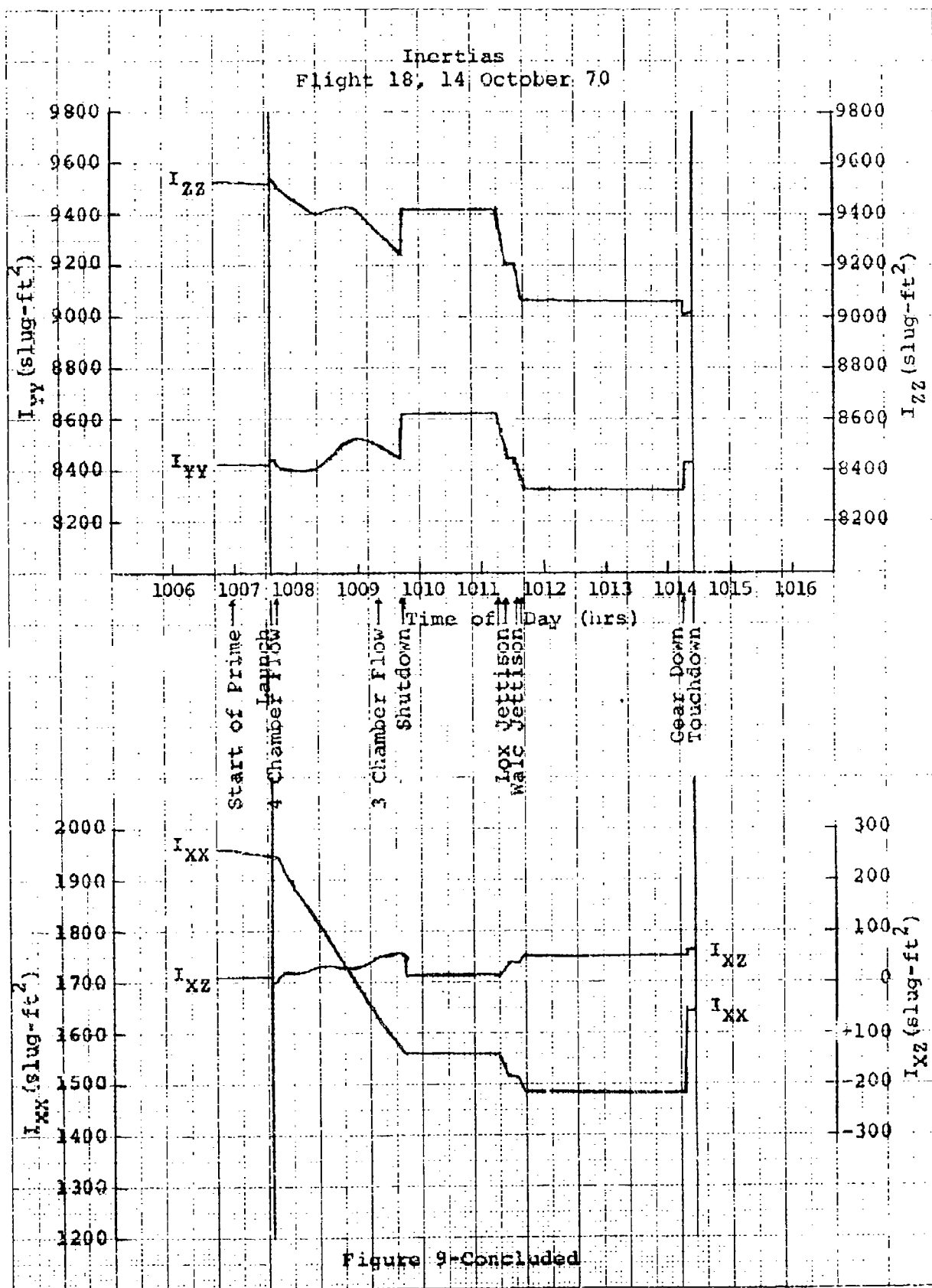


Figure 9-Gross Weight, Center of Gravity, and Inertias



Gross Weight and Center of Gravity Flight 19, 27 October 70

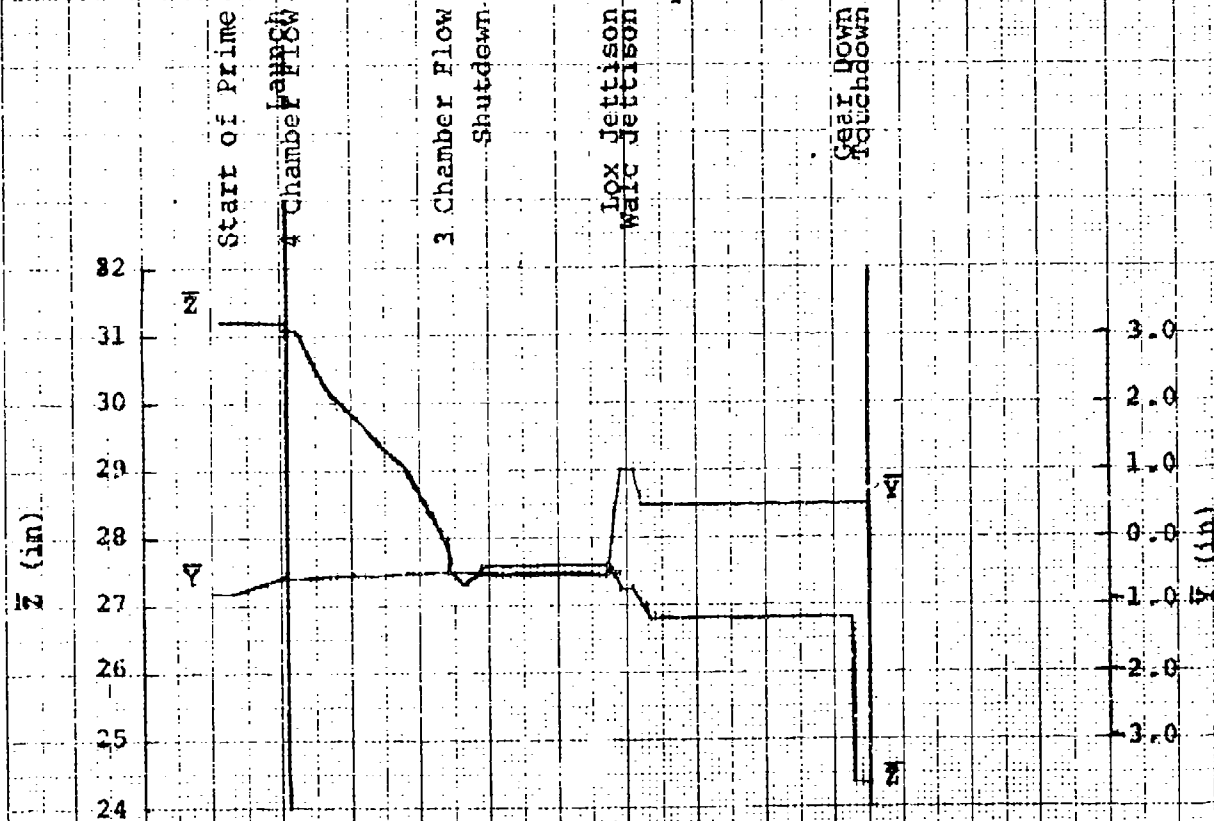
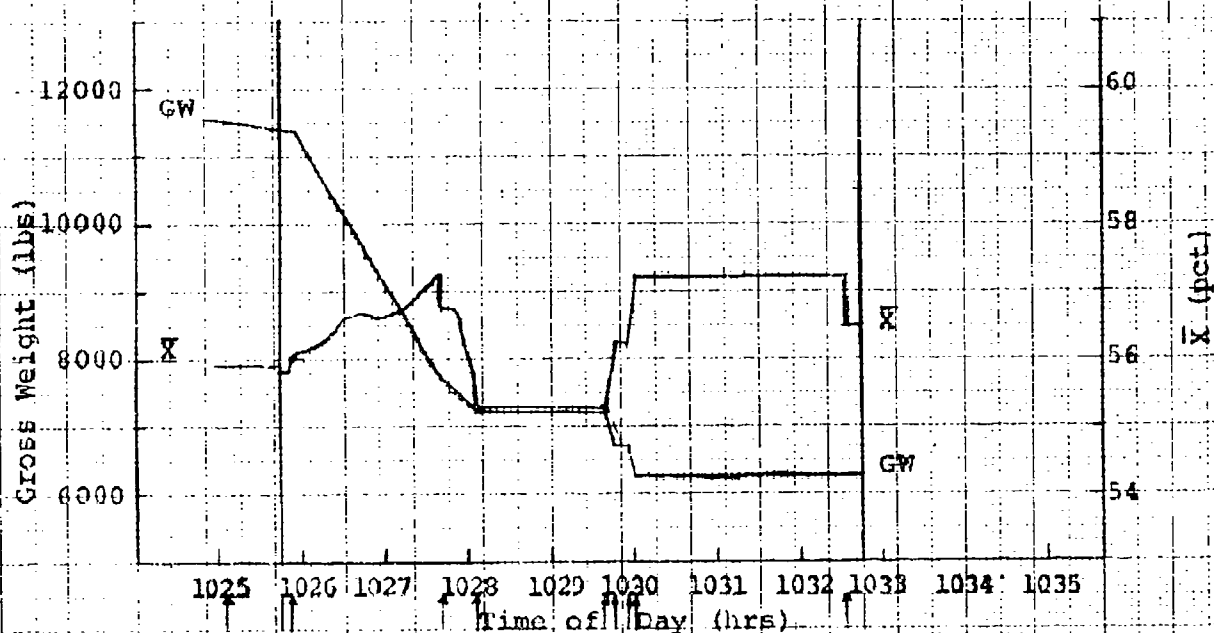
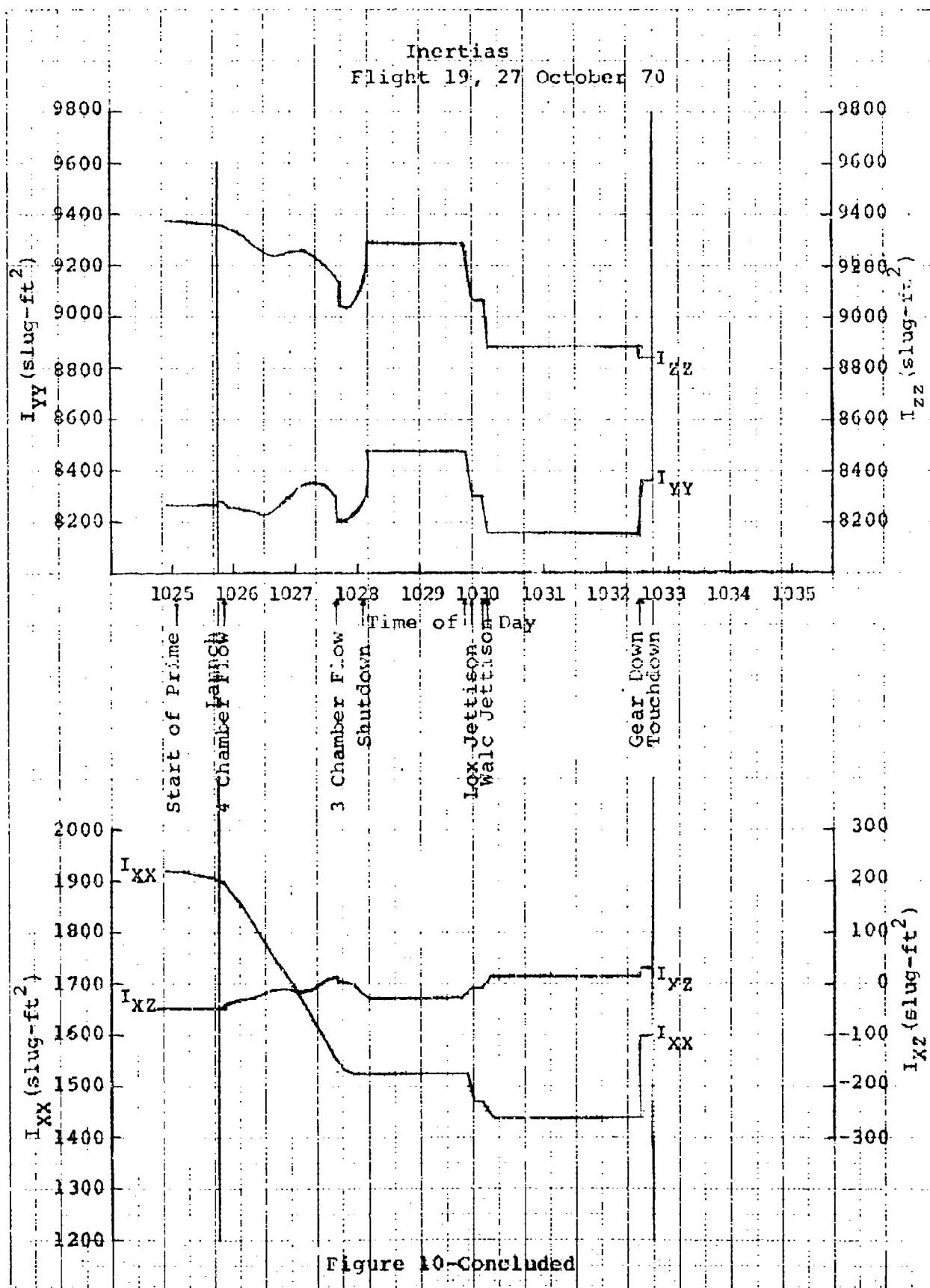


Figure 10-Gross Weight, Center of Gravity, and Inertias



Gross Weight and Center of Gravity Flight 20, 20 November 70

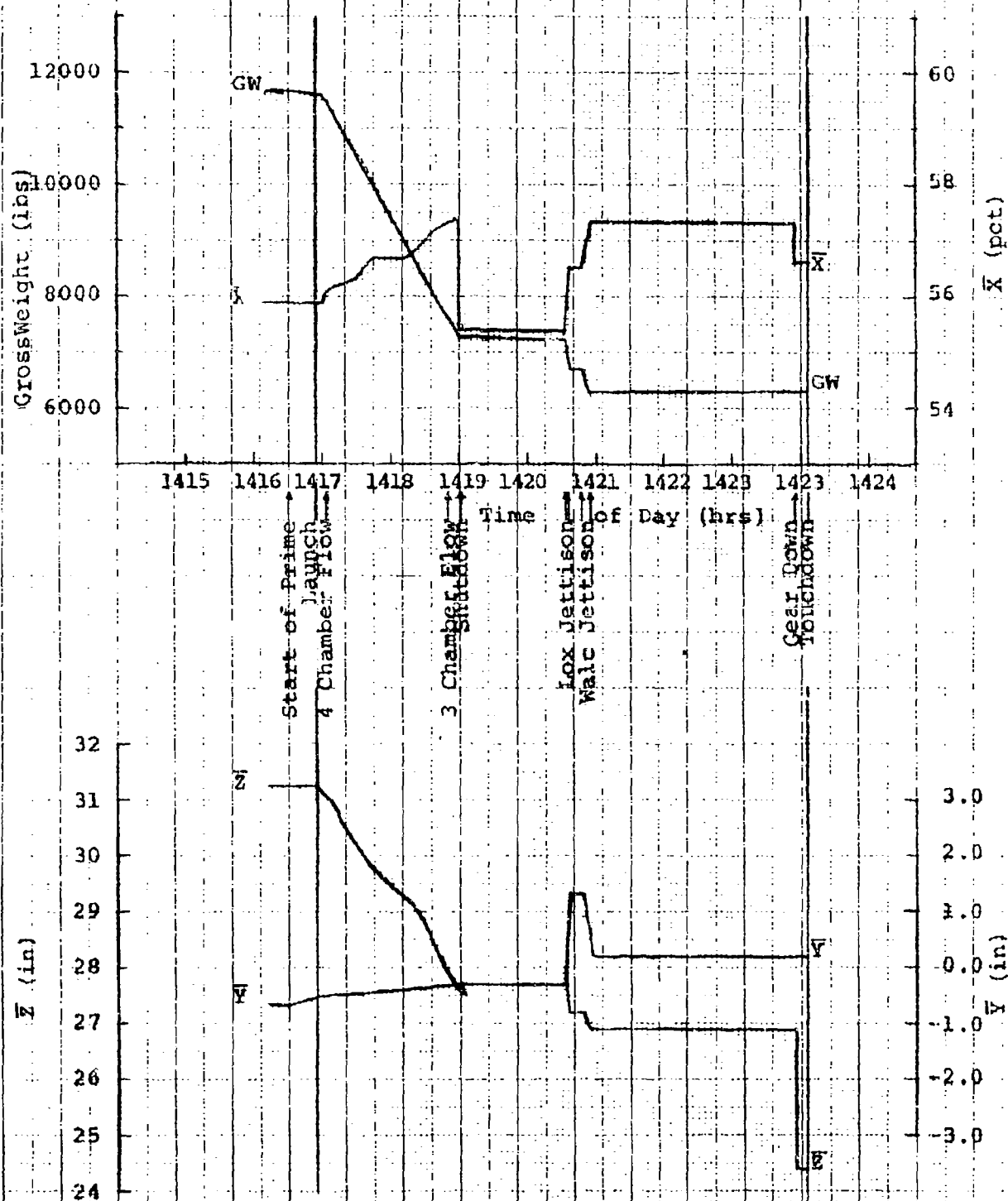
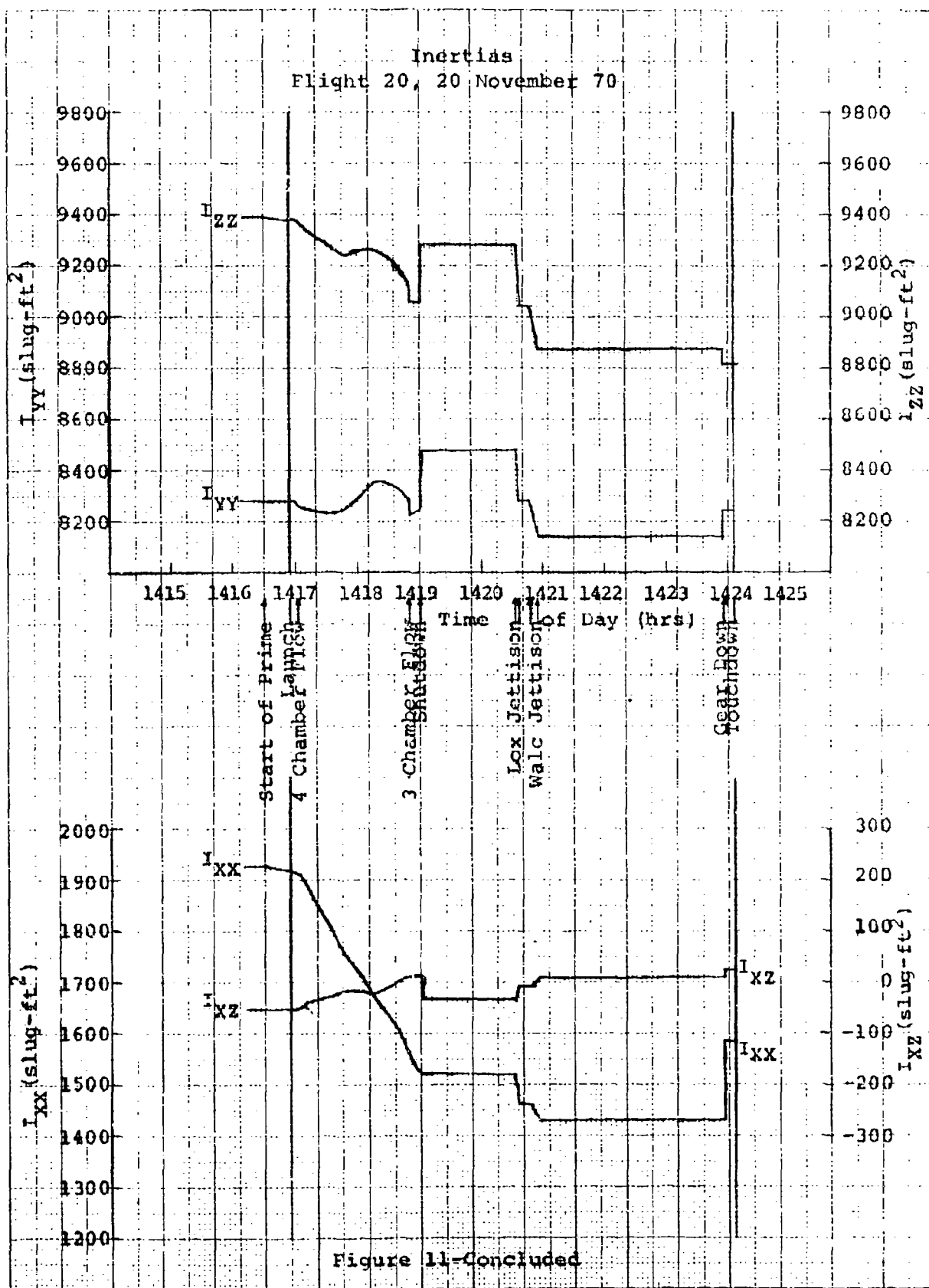
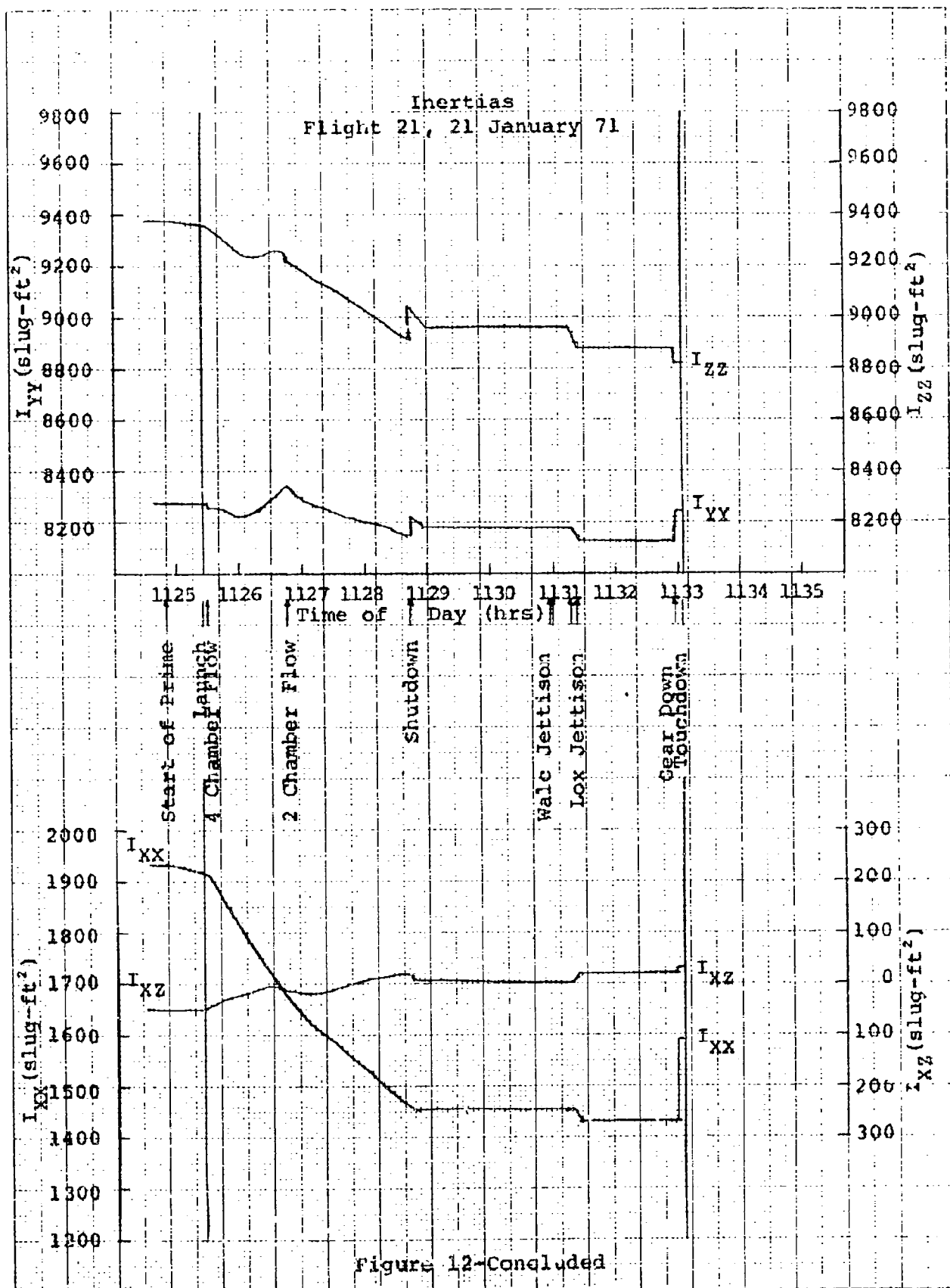


Figure 11-Gross Weight, Center of Gravity, and Inertias





Gross Weight and Center of Gravity Flight 23, 18 February 71

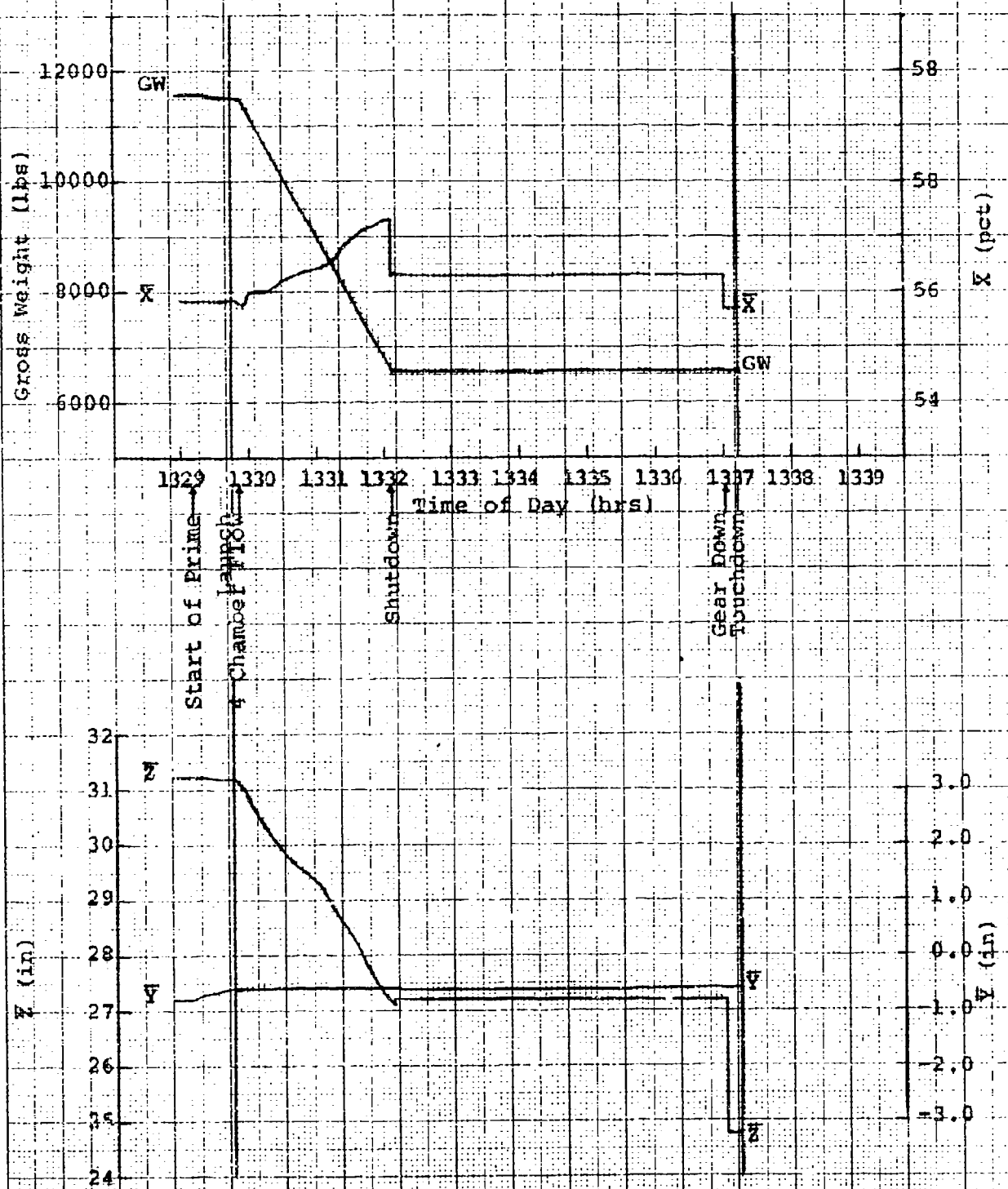


Figure 13-Gross Weight, Center of Gravity, and Inertias

Inertias Flight 23, 18 February 71

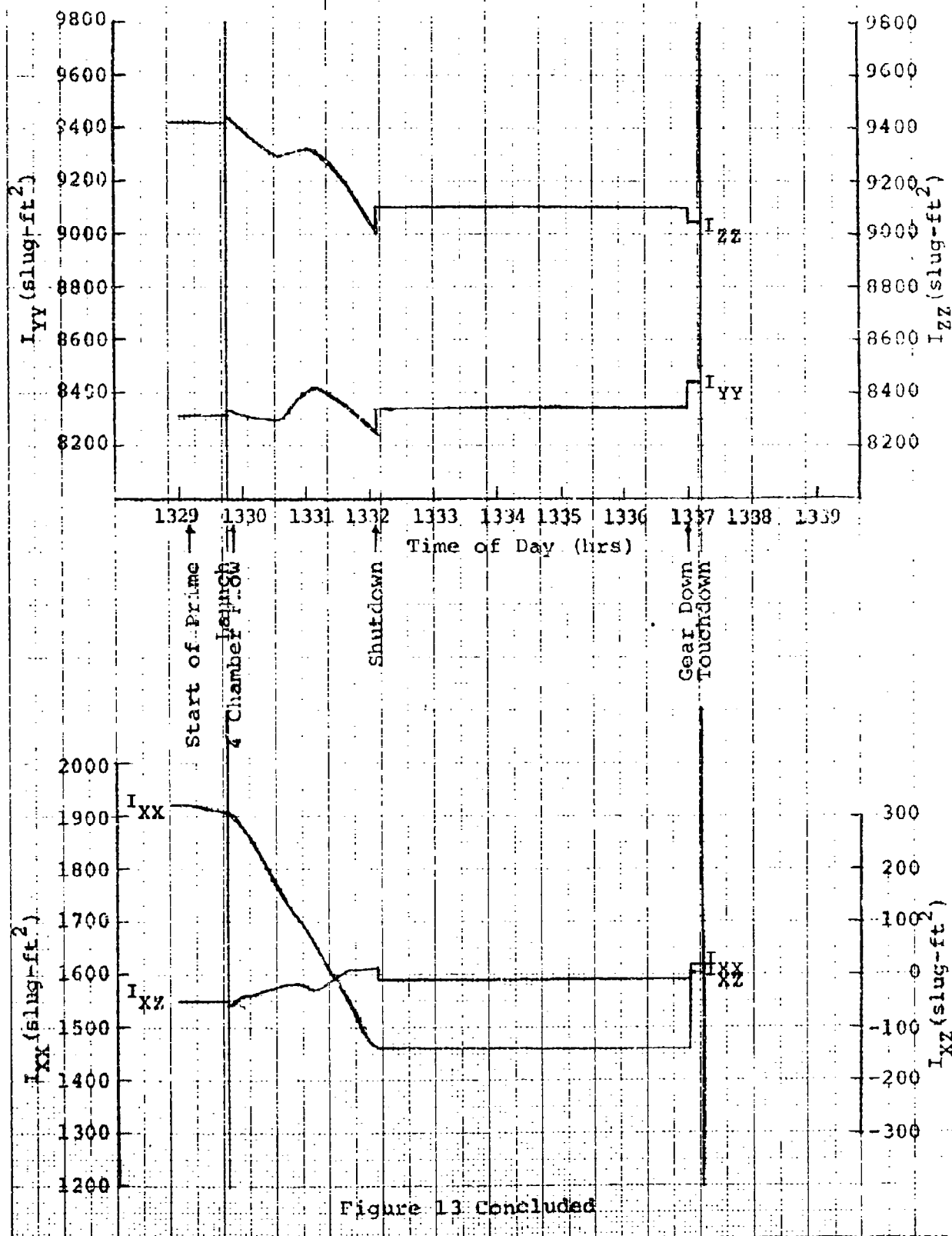


Figure 13 Concluded

Gross Weight and Center of Gravity Flight 24, 8 March 71

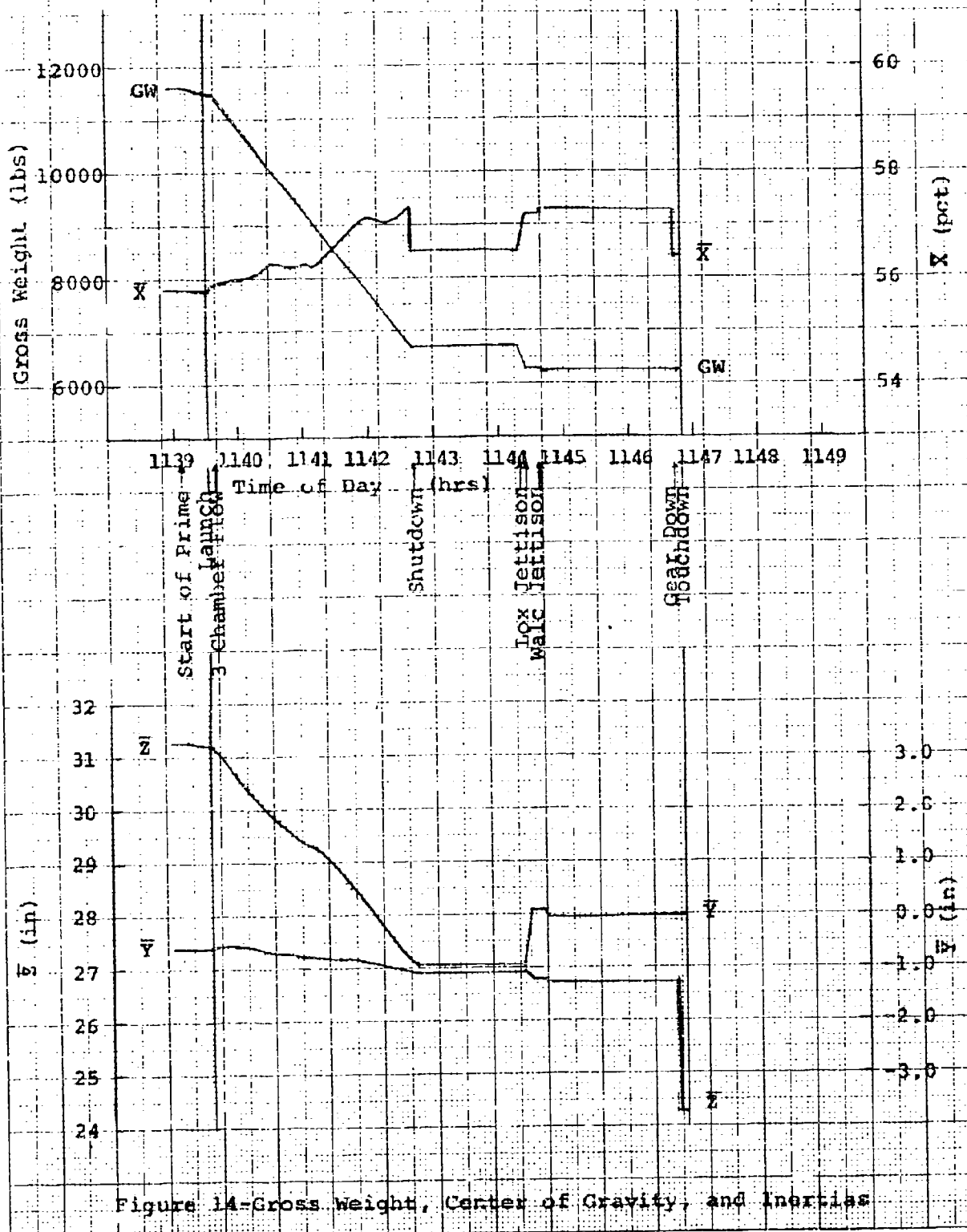


Figure 14-Gross Weight, Center of Gravity, and Inertias

Inertias Flight 24, 8 March 71

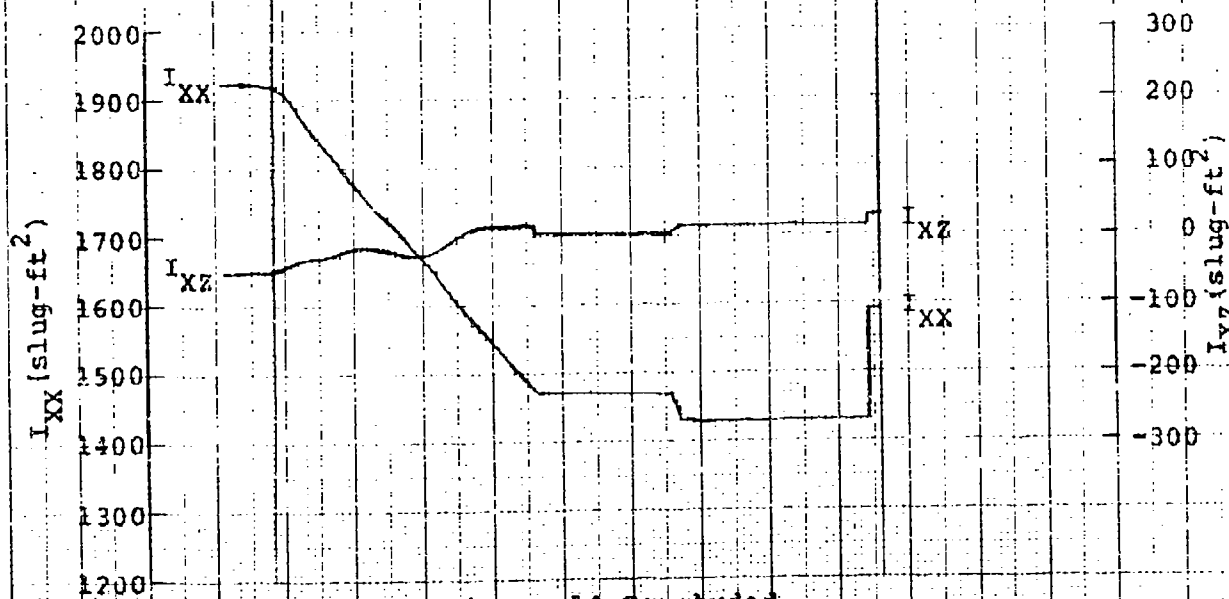
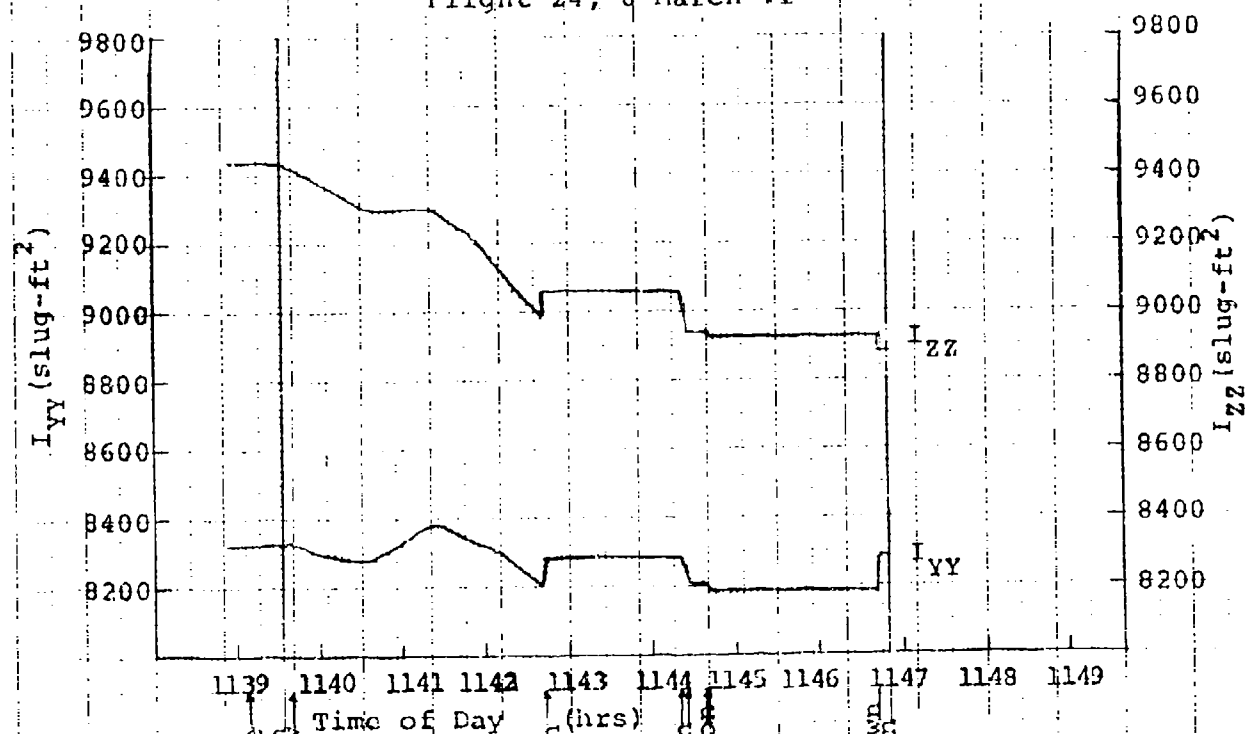


Figure 14-Concluded

Gross Weight and Center of Gravity Flight 25, 29 March 71

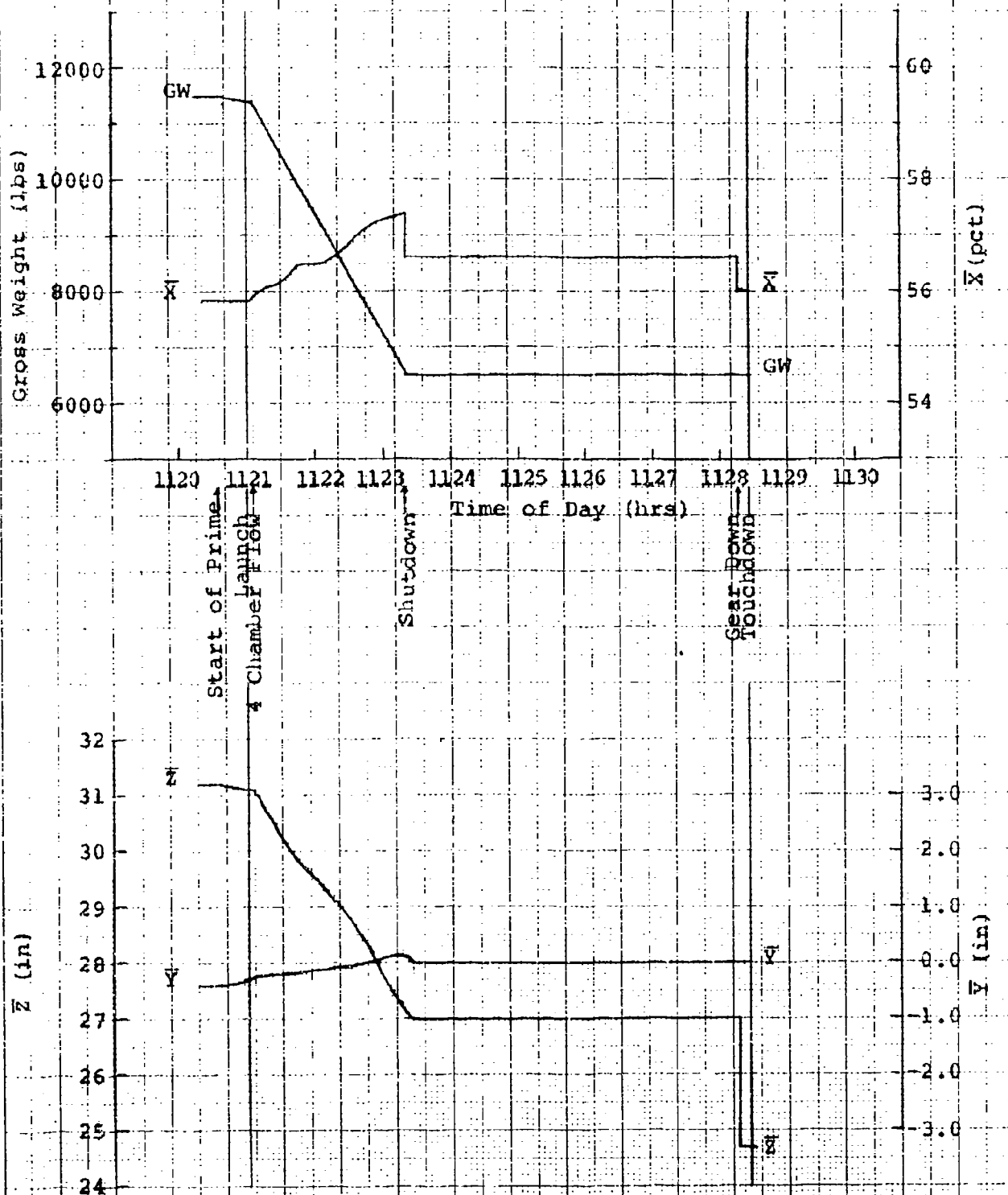


Figure 15-Gross Weight, Center of Gravity, and Inertias

Inertias Flight 25, 29 March 71

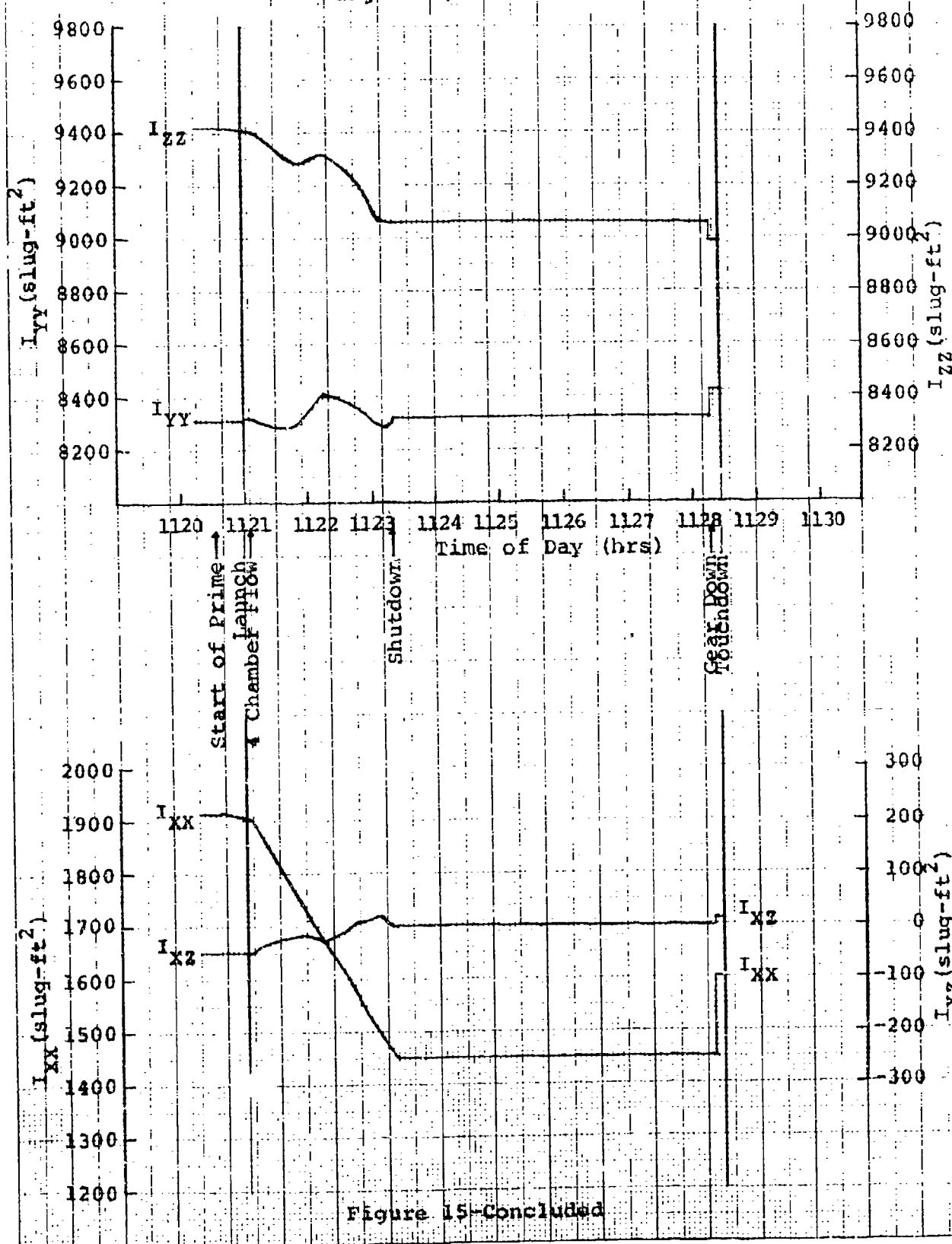


Figure 15-Concluded

Gross Weight and Center of Gravity Flight 26, 20 May 71

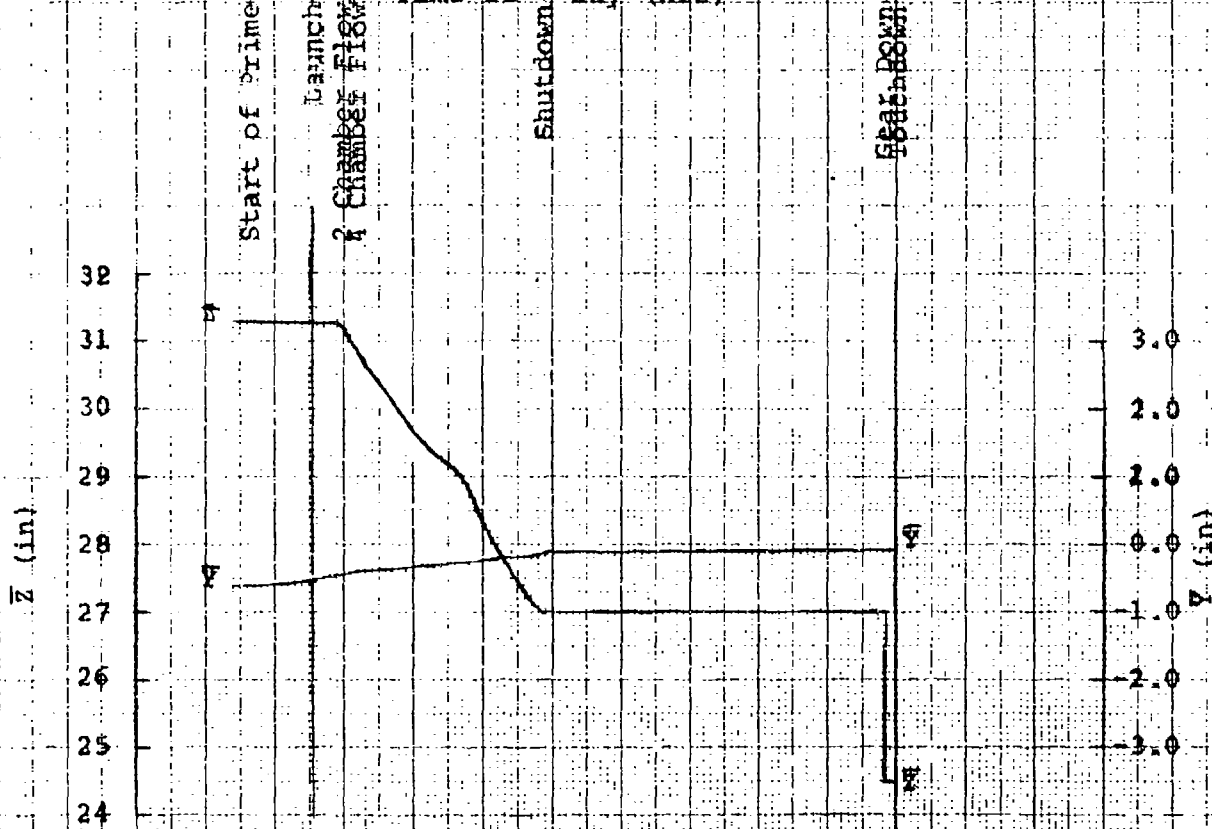
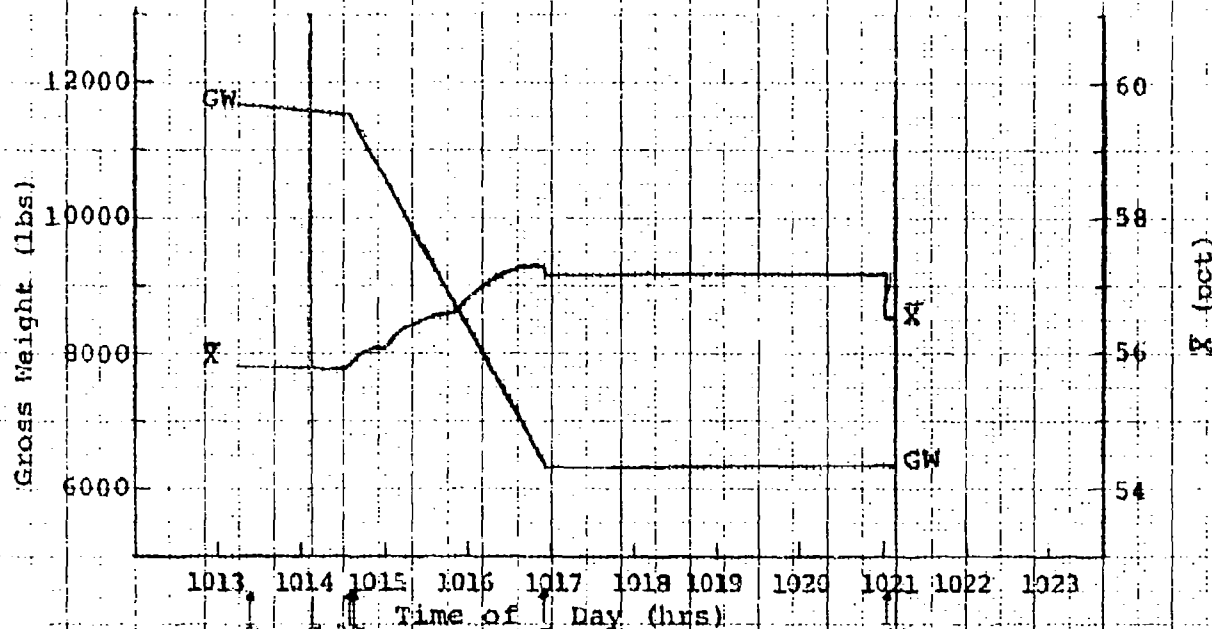
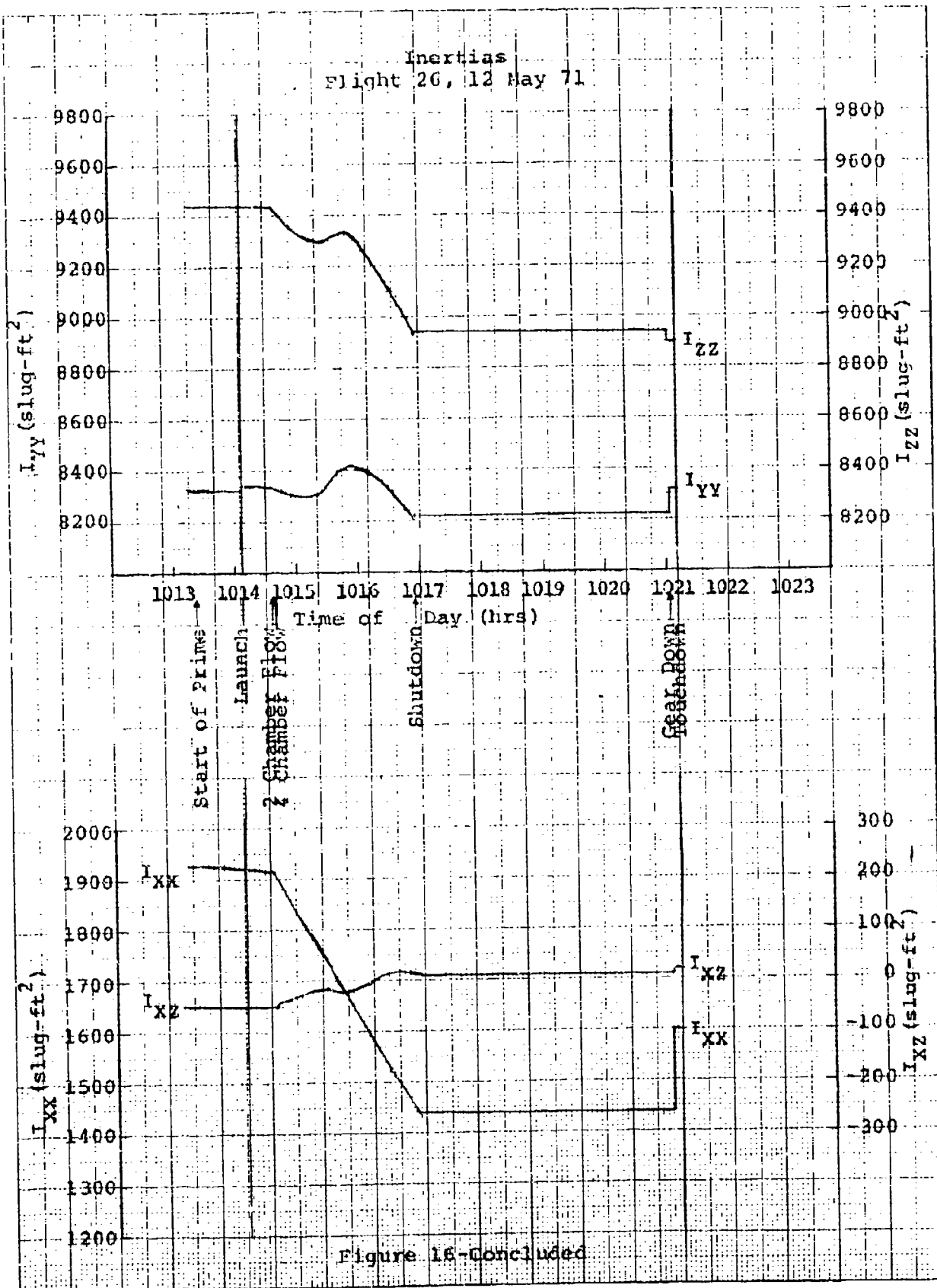


Figure 16-Gross Weight, Center of Gravity, and Inertias



Gross Weight and Center of Gravity Flight 27, 25 May 71

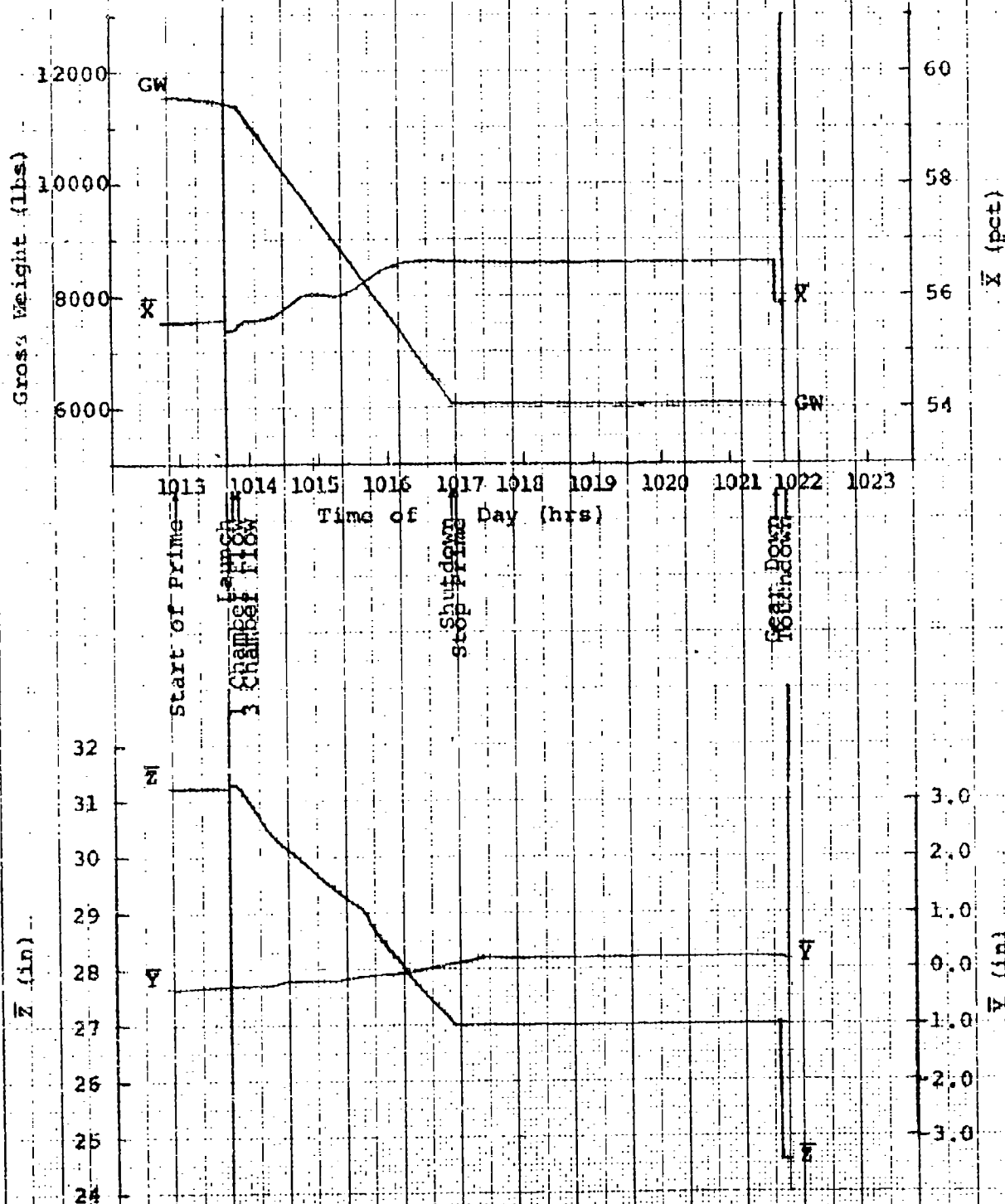


Figure 17-Gross Weight, Center of Gravity, and Inertias

Inertias
Flight 27, 25 May 71

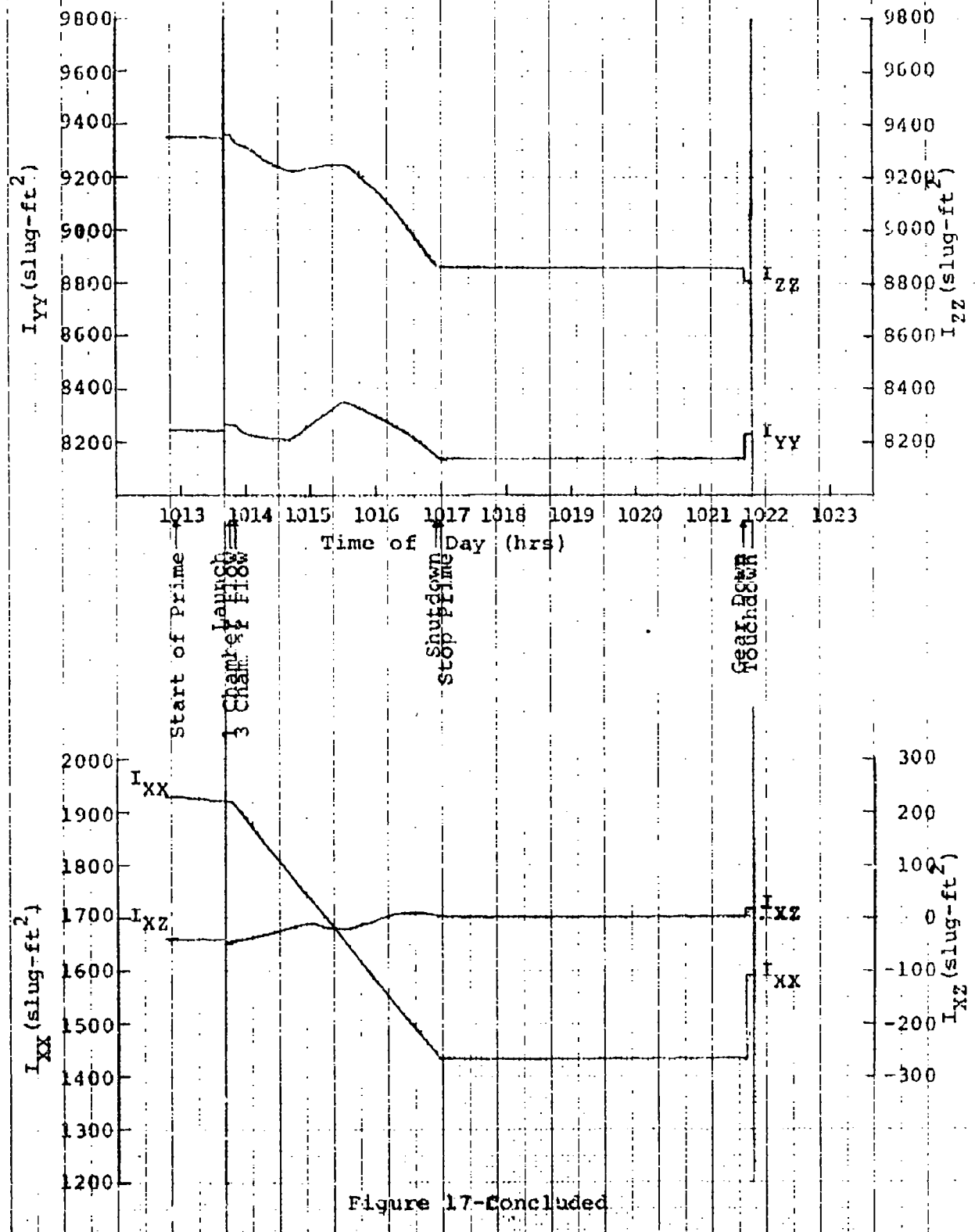


Figure 17-Concluded

Gross Weight and Center of Gravity Flight 28, 4 June 71

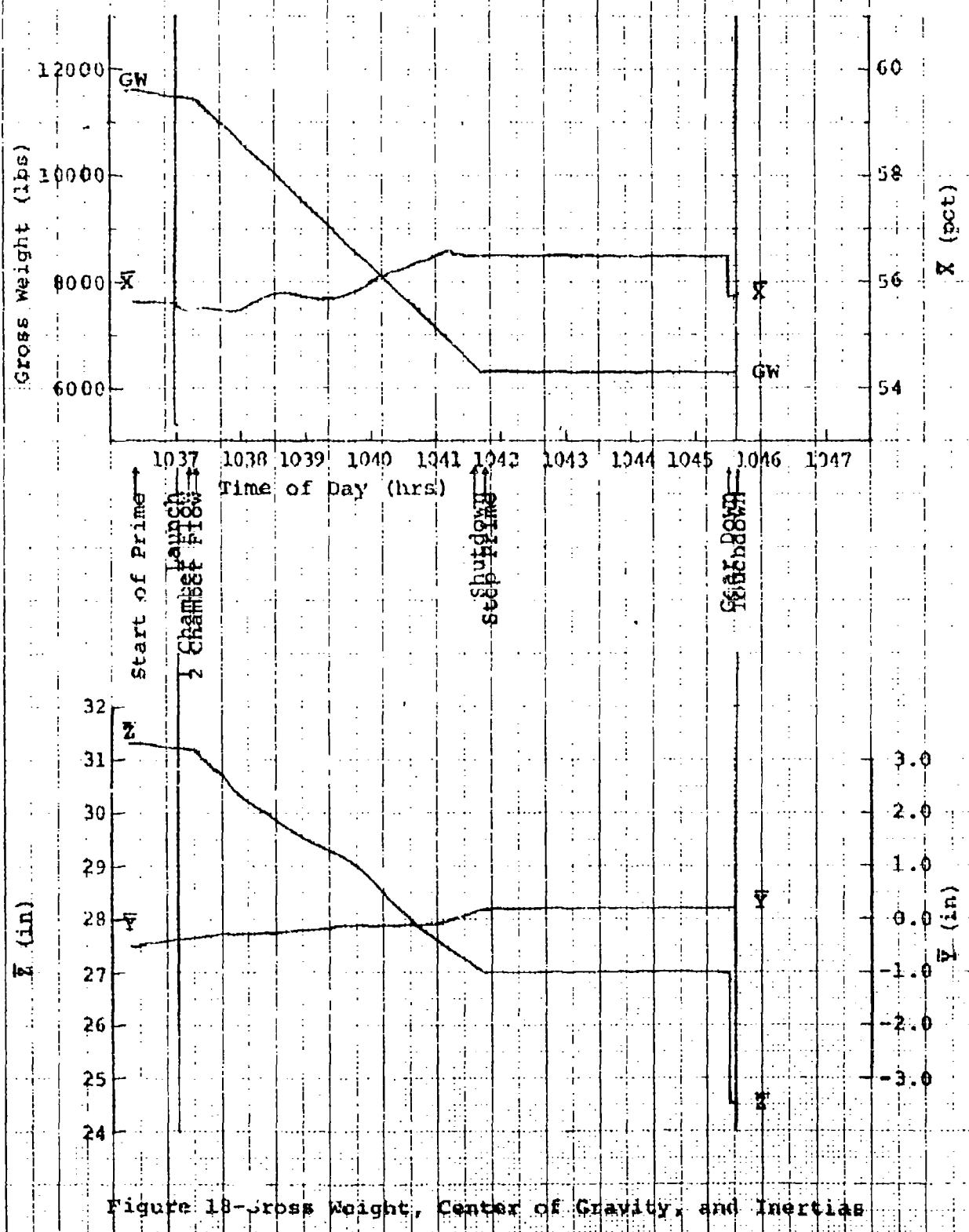
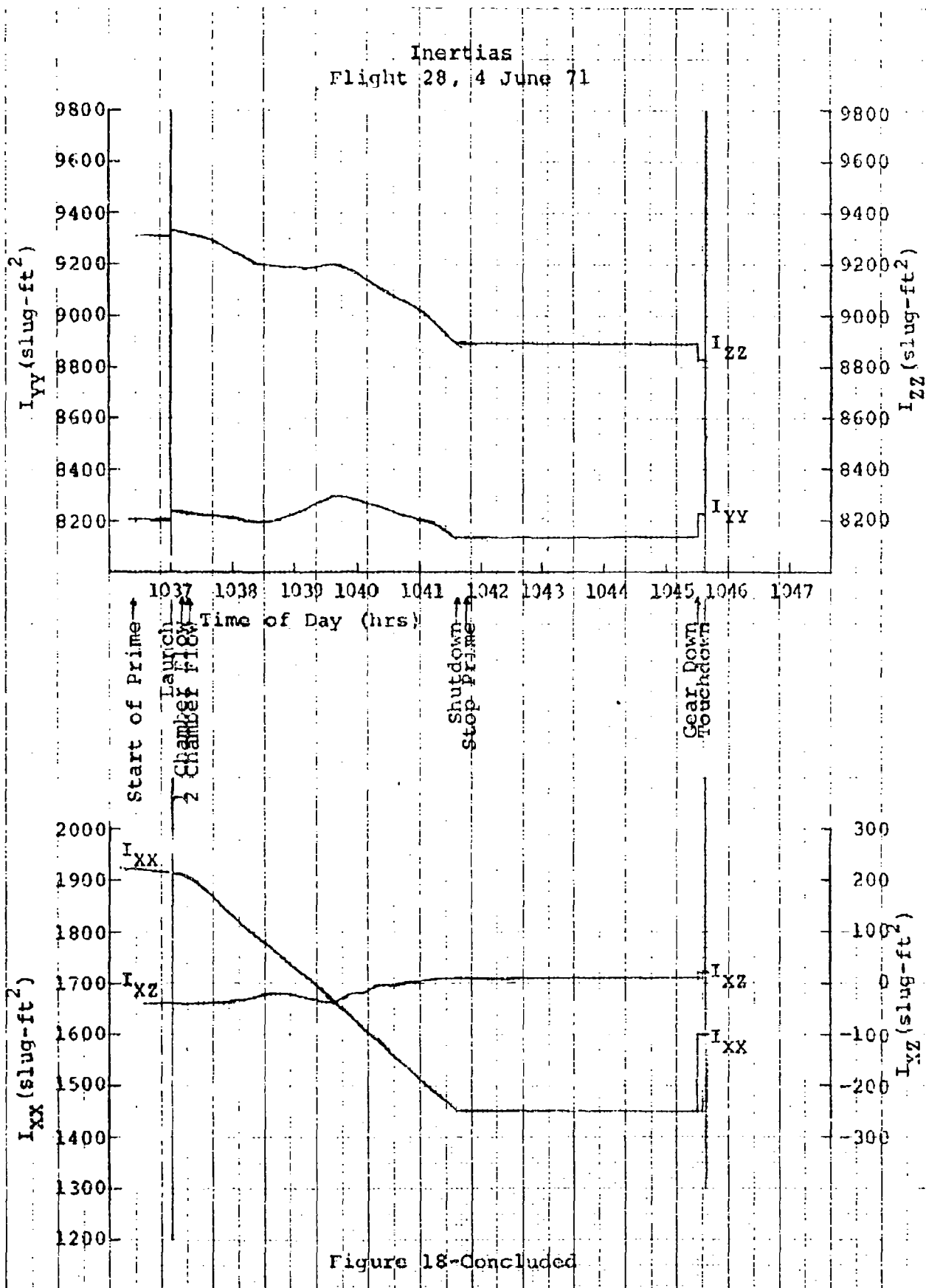


Figure 18-Gross Weight, Center of Gravity, and Inertias



APPENDIX VII

COMPARISON OF PREDICTED AND ACTUAL X-24A WEIGHTS

Starting from the baseline values, the weights and cg's were constantly updated. This tabular listing shows how the predicted updated weight compared with the actual weight measured at the AFMTC Weight and Balance Facility.

<u>Actual Weight</u> <u>(Weighing Config)</u>	<u>Predicted Wt</u> <u>(Weighing Config)</u>	<u>New Predicted</u> <u>Wt (Empty Acft</u> <u>Configuration)</u>	<u>Weight</u> <u>Error</u>
--	---	--	-------------------------------

1. 6380	(Baseline)	5927	
---------	------------	------	--

This is the baseline weighing used for the moment of inertia measurement.

2. 5956 (4 Mar 69)	5966	5917	+10 lb
-----------------------	------	------	--------

This weight was used for flights X-1 through X-3-5.

3. 6000	5972.2	6000	-27.8 lb
---------	--------	------	----------

This weighing was before flight X-3-5 but was not incorporated until flight X-4-7. This weight was used for flights X-4-7 through X-8-12.

4. 6023.0 (3 Feb 70)	5998.5	6018.0	-24.5
-------------------------	--------	--------	-------

This was used for flights X-9-14 through X-17-22.

5. 5652.7 (14 Oct 70)	5670.1	5901.35	+17.4
--------------------------	--------	---------	-------

This aircraft was weighed without hydraulic batteries on board. This empty weight was used for flight X-18-23.

6. 5893 (27 Oct 70)	5901	5893	+8.0
------------------------	------	------	------

This weight was used for flights X-19-24 through X-26-32, except for the glide flight, X-22-27.

7. 5820 (25 May 71)	5814.5	5820	-5.5
------------------------	--------	------	------

This weight was used for flights X-27-33 through X-28-34.

APPENDIX VIII

TRAPEZOIDAL APPROXIMATION OF PROPELLANT c_g AS A FUNCTION OF PROPELLANT ANGLE

Table I
cg LOCATIONS OF PROPELLANTS

Liquid Oxygen Oxidizer Tank (Empty Tank cg Position: $\bar{X}=14.7$ in., $\bar{Y}=19.5$ in., $\bar{Z}=36.5$ in.)												
Weight Fraction lb	$p=0$		$p=+15$		$p=-15$		$p=+30$		$p=-30$		$p=+60$	
	\bar{X}	\bar{Z}	\bar{X}	\bar{Z}	\bar{X}	\bar{Z}	\bar{X}	\bar{Z}	\bar{X}	\bar{Z}	\bar{X}	\bar{Z}
1.0	147.5	36.50	147.50	36.50	147.50	36.50	147.50	36.50	147.50	36.50	147.50	36.50
7/8	147.5	35.11	151.64	35.89	143.36	35.84	152.72	35.58	142.28	35.58	153.70	36.14
3/4	147.5	34.50	154.56	35.62	140.44	35.62	157.10	35.18	137.90	35.18	158.10	35.94
5/8	147.5	33.38	157.47	35.11	137.53	35.11	160.70	34.42	134.30	34.42	162.45	35.46
1/2	147.5	32.13	160.47	34.49	134.53	34.49	165.12	33.54	129.88	33.54	166.97	35.02
3/8	147.5	31.12	164.10	34.17	130.90	34.17	171.30	33.00	123.70	33.00	173.30	34.70
1/4	147.5	29.62	167.65	33.93	127.35	33.93	176.20	32.65	118.60	32.65	177.70	34.40
1/8	147.5	28.64	169.30	33.70	125.70	33.70	180.50	32.30	114.40	32.30	180.20	34.10

Water-Alcohol Fuel Tank (Empty Tank cg Position: $\bar{X}=149.5$, $\bar{Y}=14.5$, $\bar{Z}=36.50$)												
Weight Fraction lb	$p=0$		$p=+15$		$p=-15$		$p=+30$		$p=-30$		$p=+60$	
	\bar{X}	\bar{Z}	\bar{X}	\bar{Z}	\bar{X}	\bar{Z}	\bar{X}	\bar{Z}	\bar{X}	\bar{Z}	\bar{X}	\bar{Z}
1.0	149.50	36.50	149.50	36.50	149.50	36.50	149.50	36.50	149.50	36.50	149.50	36.50
7/8	142.00	35.62	147.18	36.50	145.50	36.50	147.70	35.01	144.06	35.62	148.42	35.72
3/4	141.71	35.05	144.15	36.50	142.00	36.50	146.25	34.96	139.55	35.48	147.06	35.66
5/8	139.99	35.13	139.42	36.50	137.50	36.50	140.97	35.66	135.63	35.64	140.77	36.02
1/2	131.88	36.50	131.88	36.50	131.88	36.50	131.88	36.50	131.88	36.50	131.88	36.50
3/8	131.88	34.50	131.88	36.50	130.50	36.50	136.70	34.38	128.00	33.05	138.60	34.30
1/4	131.88	32.14	139.30	36.50	127.50	36.50	142.13	33.45	124.30	32.65	144.00	33.35
1/8	131.88	29.65	146.30	36.50	124.50	36.50	147.53	32.20	118.80	32.30	146.53	31.40

HYDROGEN PEROXIDE TANK
Empty Tank cg Position
($\bar{X}=194.0$, $\bar{Y}=1.8$, $\bar{Z}=23.50$)

Weight Fraction lb	cg location $p=0$	
	\bar{X}	\bar{Z}
1.0	194.0	23.50
7/8	194.0	21.76
3/4	194.0	20.71
5/8	194.0	19.86
1/2	194.0	18.98
3/8	194.0	18.11
1/4	194.0	17.23
1/8	194.0	16.20

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4. X-24A Lifting Body Planning and Operations, FTC-TD-71-10, Air Force Flight Test Center, Edwards AFB, California, to be published.
5. X-24A Lifting Body Handling Qualities, FTC-TD-71-11, Air Force Flight Test Center, Edwards AFB, California, to be published.
6. X-24A Lifting Body Stability Derivatives, FTC-TD-71-7, Air Force Flight Test Center, Edwards AFB, California, to be published.
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10. Brandt, Jerome C., X-24A Propellant System Development and Qualification Program, FTC-TD-69-7, Air Force Flight Test Center, Edwards AFB, California, August 1969.

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5. AUTHOR(S) (First name, middle initial, last name) John P. Retelle, Captain, USAF			
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11. SUPPLEMENTARY NOTES N/A		12. SPONSORING MILITARY ACTIVITY 6510th Test Wing Edwards AFB, Calif.	
13. ABSTRACT Accurate values of weight, center of gravity, and moments of inertia were measured prior to the first flight of the X-24A lifting body. The weight, longitudinal, and lateral centers of gravity were measured at the AFFTC Weight and Balance Facility. The vertical center of gravity was measured by suspending the aircraft from a cable and determining the tilt angles as weights were added at the nose. Moments of inertia about each axis were measured by restraining the vehicle with springs and allowing it to vibrate about knife edges in the X- and Y-axes and a suspension cable in the Z-axis. These values were used as a baseline for mass data determination throughout the flight test program. A digital computer program was used to update the mass data for aircraft configuration changes and to produce time histories of mass data for powered flights, including the effects of rocket propellant flow and the changes in position of propellant in the tanks which result from accelerations on the aircraft.			

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1 NOV 65UNCLASSIFIED
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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
X-24A lifting body weight center of gravity moments of inertia mass data propellant flow						